The Non-uniqueness Problem of the Dirac Theory: "Conservative" vs "Radical" Solutions

Mayeul Arminjon

CNRS and Universités de Grenoble: UJF, Grenoble-INP

BP 53, F-38041 Grenoble cedex 9, France.

The standard curved-spacetime Dirac equation is known to be unique in a topologically simple spacetime [6, 8, 10]. Nevertheless, it has been shown recently that it leads to the generic non-uniqueness of the Hamiltonian and energy operators, including the non-uniqueness of the energy spectrum [2]. A particular manifestation of this problem had been already observed by Ryder [12]. He had found that, in the case of uniform rotation with respect to an inertial frame in the Minkowski spacetime, the presence or absence of Mashhoon's "spin-rotation coupling term" [11, 9] in the Dirac Hamiltonian depends on the choice of the tetrad field [12]. (The tetrad field determines the field of the Dirac γ^μ matrices.) Independently of that work, it had been found that, in a given coordinate system, the hermiticity of the Dirac Hamiltonian depends on the choice of the admissible field of the γ^μ matrices [1]. This led us to a general study of the non-uniqueness problem of the generally-covariant Dirac theory [2]. Indeed this problem is there already *in a Cartesian coordinate system in the Minkowski spacetime* [4], as soon as one uses one among the existing generally-covariant versions of the Dirac equation.

That problem should be solved by restricting the choice of the Dirac gamma field in a consistent way. A first, rather conservative way of adequately restricting this choice, consists in *fixing the rotation rate field associated with the orthonormal tetrad field* $(u_\alpha)_{\alpha=0,\dots,3}$, in the following way [4]. The data of a *reference frame* F fixes a four-velocity field v [7, 3]. It is natural to impose that $u_0 = v$. We show that then the spatial triad $(u_p)_{p=1,2,3}$ can only be rotating w.r.t. F. We show also that all tetrad fields (u_α), which have the same time-like vector field u_0, and for which the rotation rate tensor field $\boldsymbol{\Xi}$ of the spatial triad (u_p) is the same, give rise to equivalent Hamiltonian operators as well as to equivalent energy operators. Since the data of a reference frame F also fixes a rotation-rate field $\boldsymbol{\Omega}$ [7, 13, 4], one way of fixing the rotation rate

$\boldsymbol{\Xi}$ is therefore to impose $\boldsymbol{\Xi} = \boldsymbol{\Omega}$. Another one is to impose that $\boldsymbol{\Xi} = \mathbf{0}$. Each of these two ways provides unique Hamiltonian and energy operators.

However, this is not necessarily easy to implement and it works only in a given reference frame. A more radical solution consists in imposing that *the gamma field should change only by constant gauge transformations.* To get that situation, we are naturally led to assume that the metric can be put in a space-isotropic diagonal form. When this is the case, we show that it distinguishes a preferred reference frame—unless there is no gravitation, i.e., unless the Minkowski spacetime is considered. We show that by defining the gamma field from the "diagonal tetrad" in any chart in which the metric has that form, a unique Hamiltonian operator as well as a unique energy operator are obtained in any reference frame [5]. We discuss the application of the different solutions proposed to the case of a uniformly rotating frame in the Minkowski spacetime. In particular, we examine the compatibility of these solutions with the prediction by Mashhoon of a "spin-rotation coupling".

References

[1] Arminjon M. and Reifler F., *Basic Quantum Mechanics for Three Dirac Equations in a Curved Spacetime*, Braz. J. Phys. **40** (2010) 242–255, [arXiv:0807.0570 (gr-qc)].

[2] Arminjon M. and Reifler F., *A Non-Uniqueness Problem of the Dirac Theory in a Curved Spacetime*, Ann. Phys. (Berlin) **523** (2011) 531–551. [arXiv:0905.3686 (gr-qc)]

[3] Arminjon M. and Reifler F., *General Reference Frames and their Associated Space Manifolds*, Int. J. Geom. Meth. Mod. Phys. **8** (2011) 155–165. [arXiv:1003.3521 (gr-qc)]

[4] Arminjon M., *A Solution of the Non-Uniqueness Problem of the Dirac Hamiltonian and Energy Operators*, Ann. Phys. (Berlin) **523** (2011) 1008–1028. [Pre-peer-review version: arXiv:1107.4556 (gr-qc)].

[5] Arminjon M., *A Simpler Solution of the Non-Uniqueness Problem of the Dirac Theory*, submitted for publication.

[6] Brill D. and Wheeler J., *Interaction of Neutrinos and Gravitational Fields*, Rev. Modern Phys. **29** (1957) 465–479. Erratum: Rev. Modern Phys. **33** (1961) 623–624.

[7] Cattaneo C., *General Relativity: Relative Standard Mass, Momentum, Energy and Gravitational Field in a General System of Reference*, Il Nuovo Cimento **10** (1958) 318–337.

[8] Chapman T. and Leiter D., *On the Generally Covariant Dirac Equation*, Am. J. Phys. **44** (1976) 858–862.

[9] Hehl F. and Ni W., *Inertial Effects of a Dirac Particle*, Phys. Rev. D **42** (1990) 2045–2048.

[10] Isham C., *Spinor Fields in Four Dimensional Space-Time*, Proc. Roy. Soc. London A **364** (1978) 591–599.

[11] Mashhoon B., *Neutron Interferometry in a Rotating Frame of Reference*, Phys. Rev. Lett. **61** (1988) 2639–2642.

[12] Ryder L., *Spin-Rotation Coupling and Fermi-Walker Transport*, Gen. Relativ. Gravit. **40** (2008) 1111–1115.

[13] von Weyssenhof J., *Metrisches Feld und Gravitationsfeld*, Bull. Acad. Polon. Sci. A **252** (1937). (Quoted by Cattaneo [7].)