Lie Groups, Differential Equations, and Moving Frames

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ABSTRACT

Although symmetry is pervasive in nature and art, the underlying mathematical concept of a "group" was formulated less than two hundred years ago. Lagrange and Galois were motivated by the symmetry of solutions of polynomial equations. A half century later, the Norwegian mathematician Sophus Lie first applied symmetry methods to differential equations, and thereby created the theory of continuous or Lie groups, which have, in the ensuing century, permeated mathematics and its applications.

In this series of lectures, I will begin by introducing the theory of Lie groups and discuss their applications to differential equations, including methods for computing symmetries and conservation laws, integration of ordinary differential equations, and determination of explicit solutions of partial differential equations.

I will then present a new approach to the geometric method of moving frames. The method is surprisingly elementary, completely algorithmic, and can be readily applied to completely general Lie group (and even pseudo-group) actions, leading to a remarkably wide range of new results and applications in geometry, algebra, differential equations, the calculus of variations, computer vision, and geometric integration. Particular topics include classification and syzygies of differential invariants and joint invariants, computation of invariant variational problems and invariant differential equations, equivalence, symmetry and rigidity properties of submanifolds, applications to object recognition in computer vision, and the design of symmetry-preserving numerical algorithms.