On Symmetry Reduction of Some P(1,4)-invariant Differential Equations

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Abstract

The development of theoretical and mathematical physics has required various extensions of the four-dimensional Minkowski space $M(1,3)$ and, correspondingly, various extensions of the Poincaré group $P(1,3)$. The natural extension of this group is the generalized Poincaré group $P(1,4)$. The group $P(1,4)$ is the group of rotations and translations of the five-dimensional Minkowski space $M(1,4)$.

The papers [1, 2] are devoted to the symmetry reduction of some differential equations in the spaces $M(1,3)\times R(u)$ and $M(1,4)\times R(u)$, which are invariant with respect to the Poincaré group $P(1,4)$. Here, and in what follows, $R(u)$ is the real number axis of the dependent variable u. To perform this reduction, we have used functional bases of invariants of nonconjugate subgroups of the group $P(1,4)$. The details on the symmetry reduction methods can be found, for example, in [3, 4, 5].

However, it turned out that the reduced equations, obtained with the help of nonconjugate subalgebras of the Lie algebra of the group $P(1,4)$ of the given rank, were of different types. It means that using only the rank of those nonconjugate subalgebras, we cannot explain differences in the properties of the reduced equations.

It should be noted that Grundland, Harnad, and Winternitz [6] were the first to point out and to try to investigate this fact.

It is known that the nonconjugate subalgebras of the Lie algebra of the group $P(1,4)$ of the same rank may have different structural properties. Therefore, to explain some of the differences in the properties of the above mentioned reduced equations, we suggest to try to investigate the connections between structural properties of nonconjugate subalgebras of the same rank of the Lie algebra of the group $P(1,4)$ and the properties of the reduced equations corresponding to them.

Until now, we have classified the functional bases of invariants in the space $M(1,3)\times R(u)$ of one, two, and three-dimensional non-conjugate subalgebras of the Lie algebra of the group $P(1,4)$ using the classification [7] of these subalgebras. In other words, we have established a connection between the classification of one, two, and three-dimensional nonconjugate subalgebras of the Lie algebra of the group $P(1,4)$ and their

invariants in the space $M(1,3) \times R(u)$.

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