Integrable Systems Arising from Motions of Curves on $S^2(R)$ and $S^3(R)$

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ABSTRACT

Motion of curves lead to nonlinear differential equations which are often integrable. They are also intimately connected to the dynamics of Heisenberg ferromagnetic equations (HFE) and other integrable systems through geometric and gauge symmetric connections/equivalence. Here we point out the fact that a more general situation in which the curves evolve in the presence of additional self consistent vector potentials can lead to interesting generalized HFE with self consistent potentials or soliton equations with self consistent potentials. We obtain the general form of the evolution equations of underlying curves and report specific examples of generalized HFE and other integrable systems. These include principal chiral model and various generalized HFE in 1+1 dimensions and their geometrically equivalent generalized nonlinear Schrödinger (NLS) family of equations, including Hirota-Maxwell-Bloch equations, all in the presence of self consistent potential fields. The associated gauge equivalent Lax pairs are also presented to confirm their integrability.