Coupled Nonlinear Schroedinger Equations: Polarization, Phase Difference, Dispersion and Nonlinearity, Quasi-Particle Dynamics

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ABSTRACT

For a system of coupled nonlinear Schroedinger equations (SCNLSE) both through linear and nonlinear terms, we investigate numerically the head-on and taking-over collision dynamics of polarized solitons. In the case of general elliptic polarization, analytical solutions for the shapes of steadily propagating solitons are not available, and we develop an auxiliary numerical algorithm for finding the initial shape. We use a superposition of polarized solitons as the initial condition for investigating the soliton dynamics (sech-like as well as general form). We consider the interactions with and without cross-modulation, with stationary shapes and with breathers. For general nontrivial cross-modulation, a jump in the polarization angles of the solitons takes place after the collision (polarization shock). For moderate and large values of the nonlinear coupling parameter, additional solitons are created during the collision of the initial ones.

We also find that, depending on the initial phases of the solitons, the polarizations of the system of solitons after the collision change, even for trivial cross- modulation. This sets the limits of practical validity of the celebrated Manakov solution. In the majority of cases the solitons survive the interaction, preserving approximately their phase speeds and the main effect is the change of individual polarization but the total net polarization of the system is conserved. However, in some intervals for the initial phase difference, the interaction is ostensibly inelastic: either one of the solitons virtually disappears, or additional solitons are born after the interaction.

In the case of pure linear coupling the Manakov system is enriched by a linear coupling term with complex-valued coeffcient and the individual solitons are actually either blowing or dispersing breathers. The momentum of the individual quasi-particles (QPs) is conserved while the masses of the individual QPs oscillate, but the sum of the masses for the two QPs is constant. Respectively, the total energy oscillates during one period of the

breathing, but the average over the period is conserved. The individual and total polarization angles for the two QPs oscillate with different periods before and after the interaction. This extends to the case of breathing our earlier results about the conservation of the total polarization for the interaction of non-breathing solitons.

The results of this work elucidate the role of the linear and nonlinear couplings, the initial phase, and the initial polarization on the interaction dynamics of soliton systems in SCNLSE. They support the validity of the so-called Boussinesq paradigm. Since the Manakov system loses its full integrability when the nontrivial nonlinear coupling is present, the approach for its study is numerical. All the numerical experiments and computer simulations are implemented by using a fully conservative difference scheme in complex arithmetic.