Covariant Quantization Rules

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ABSTRACT

We consider covariant quantization rules for nonsingular (theories without constraints in Hamiltonian formulation) finite dimensional systems with flat and curved configuration spaces. In the beginning, we construct a family of covariant under the transformations from the group $O(r,s)$ quantizations in flat spaces and Cortezian coordinates. This family is parametrized by a function $\omega(\theta)$, $\theta \in (1,0)$, which describes an ambiguity of the quantization. We generalize these results to covariant quantizations of systems also with flat configuration spaces but already described by arbitrary curvalinear coordinates. Then we represent a minimal covariant under general coordinate transformations generalization of the quantizations in flat spaces to curved spaces. This generalization is parametrized by the same function $\omega(\theta)$. After that, we construct an extended family of covariant under general coordinate transformations quantizations in a curved space. This family is already parametrized by two functions, the previous one $\omega(\theta)$ and by an additional functon $\Theta(x,\xi)$. The minimal generalization represents a part at $\Theta = 1$ of the extended family of quantizations. We study constructed quantizations in detail, prooving their consistency, covariance and the fulfillment of the correspondence principle for them. As physical applications, we consider a quantization of a non-relativistic particle moving in a curved space, discussing the problem of a quantum potential. Applying the covariant quantizations in flat spaces to an old problem of constructing quantum Hamiltonian in Polar coordinates, we directly obtain a correct result.