Aspects of Noncommutative Dynamics

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Abstract

Formulating classical and quantum dynamics on noncomutative spaces has been expected for a long time to offer a solution to the problem of infinities in classical and especially quantum field theory. This hope has not been confirmed for the time being. Yet a number of surprising effects has emerged, allowing us to test in a workable context the limits of present theories. Noncommutative spaces forced us to enlarge our view of fundamental concepts like gauge invariance, locality, causality, dimensional reduction, or even the relation between the so-called first and second quantized theories.

All these, and more, make noncommutative theories an excellent theoretical laboratory, in which fundamental geometric concepts and quantization can be extended in a new but controlable and often quite suggestive set-up.

On the more practical side, noncomutativity offers some different technical solutions, like somewhat different perturbative expansions or a natural discretization of space. This last point provides a ready-made solution to the half-century old problem of putting fermions on a lattice.

It is important to distinguish between two types of noncommutativity which are different yet shed light on each other. These will be called "particle noncommutativity" and "field noncommutativity"; the presentation will be structured around these two concepts, as detailed in the following.

Outline

I) Particle Noncommutativity (NC)

1. Noncommutativity via special limits (large magnetic fields, string NC)
2. Direct formulation in phase space; dimensional reduction
3. Variational formulation and classical Zitterbewegung
4. Breaking and restoration of gauge invariance
5. Variable noncommutativity and the Feynman problem
6. Quantization: Schrödinger formulation examples and generalized wave mechanics
7. Quantization: Canonical formalism versus path integrals and a pleasant surprise
8. Experimental signals - some instructive examples

II) Field Noncommutativity

1. A puzzle in quantization: phase operator and a "mistake" of Dirac
2. Conjugate variables and a precise counting of the degrees of freedom
3. Consistent (non-)locality; dipoles and their interactions
4. Quantization without tears
5. Causality and effective theories
6. NC-induced discretization of space
7. Generalized Noether theorem and a holographic interpretation of states
8. Exact solutions and classical finiteness
9. Heisenberg limit of $SU(2)$ noncommutativity
10. More exact solutions and their interpretation
11. Snyder NC and (anti)deSitter spaces
12. Klein and Cartan, Groups and Geometries

The hope is to advance at a rate of about four of the above topics per lecture.