# Dual $\kappa$-Minkowski spaces and $\kappa$-Poincaré algebras from Yang model and their Weyl realizations

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# Realizations of Lie algebras

Let $\mathfrak{g}$ be a finite dimensional Lie algebra over a field $\mathbb{K}$ of characteristic 0.
Let $\{X_\mu \mid 1 \leq \mu \leq n\}$ be a ordered basis of $\mathfrak{g}$ satisfying the Lie bracket relations

$$[X_\mu, X_\nu] = \sum_{\alpha=1}^{n} C_{\mu\nu\alpha} X_\alpha, \tag{1}$$

where $C_{\mu\nu\alpha}$ are the structure constants of $\mathfrak{g}$.
The structure constants obey $C_{\mu\nu\alpha} = -C_{\nu\mu\alpha}$ and the Jacobi identity:

$$\sum_{\rho=1}^{n} (C_{\mu\alpha\rho} C_{\rho\beta\nu} + C_{\alpha\beta\rho} C_{\rho\mu\nu} + C_{\beta\mu\rho} C_{\rho\alpha\nu}) = 0. \tag{2}$$

Let $\mathfrak{g}_h$ be a Lie algebra over a field $\mathbb{K}$ defined by

$$[X_\mu, X_\nu] = \sum_{\alpha=1}^{n} C_{\mu\nu\alpha}(h) X_\alpha, \quad 1 \leq \mu, \nu \leq n, \tag{3}$$

such that $\lim_{h \to 0} C_{\mu\nu\alpha}(h) = 0$.

Let $\mathcal{A}_n$ be the n-th Weyl-Heisenberg algebra generated by $x_i, \partial_i, 1 \leq i \leq n$, satisfying

$$[x_i, x_j] = [\partial_i, \partial_j] = 0, \ [\partial_i, x_j] = \delta_{ij}. \tag{4}$$

Let $\hat{\mathcal{A}}_n$ denote the semicompleted Weyl-Heisenberg algebra with respect to the degree of differential operators $\partial_\mu, \ \mu = 1, 2, \dots, n$.

## Definition

A realization of the Lie algebra (3) is a Lie algebra monomorphism $\varphi \colon \mathfrak{g}_h \to \hat{\mathcal{A}}_n$ defined on the basis of $\mathfrak{g}$

$$\varphi(X_\mu) = \sum_{\alpha=1}^{n} x_\alpha \varphi_{\alpha\mu}(\partial) \tag{5}$$

where $\varphi_{\alpha\mu}(\partial)$ is formal power series in $\partial_1, \dots \partial_n$ depending on $h$ such that $\lim_{h \to 0} \varphi_{\alpha\mu}(\partial) = \delta_{\alpha\mu}$.

We introduce the notation

$$\hat{x}_\mu = \sum_{\alpha=1}^{n} x_\alpha \varphi_{\alpha\mu}(\partial). \tag{6}$$

Let $\hat{\mathcal{X}} = \tilde{\varphi}(U(\mathfrak{g}_h))$. The algebra $\hat{\mathcal{X}}$ is generated by $\hat{x}_1, \dots, \hat{x}_n$ which satisfying the commutation relations:

$$[\hat{x}_\mu, \hat{x}_\nu] = \sum_{\alpha=1}^{n} C_{\mu\nu\alpha} \hat{x}_\alpha. \tag{7}$$

## Proposition 1

The generators $\hat{x}_\mu = \sum_{i=\alpha}^{n} x_\alpha \varphi_{\alpha\mu}(\partial)$, $1 \leq \mu \leq n$, satisfy the commutation relations (7) if and only if the functions $\varphi_{\alpha\mu}$ are solutions of the following system of PDEs:

$$\sum_{\alpha=1}^{n} \left( \frac{\partial \varphi_{\lambda\mu}}{\partial \partial_\alpha} \varphi_{\alpha\nu} - \frac{\partial \varphi_{\lambda\nu}}{\partial \partial_\alpha} \varphi_{\alpha\mu} \right) = \sum_{\alpha=1}^{n} C_{\mu\nu\alpha} \varphi_{\lambda\alpha}, \ \lambda, \mu, \nu = 1, 2, \dots, n. \tag{8}$$

# The Weyl symmetric realization of Lie algebras

Let $\mathcal{X} = \mathbb{K}[x_1, \ldots, x_n]$. We introduce the action $\triangleright \colon \mathcal{A}_n \otimes \mathcal{X} \to \mathcal{X},\ a \otimes f \to a \triangleright f$ defined by:

1. $x_\mu \triangleright f = x_\mu f$,
2. $\partial_\mu \triangleright f = \frac{\partial f}{\partial x_\mu}$,
3. $(ab) \triangleright f = a \triangleright (b \triangleright f),\ \forall a, b \in \mathcal{A}_n$ i $f \in \mathcal{X}$.

For a given realization $\varphi \colon U(\mathfrak{g}_h) \to \hat{\mathcal{A}}_n$ define the vector space isomorphism $\Omega_\varphi \colon \hat{\mathcal{X}} \to \mathcal{X}$ :

$$\Omega_\varphi(\hat{f}) = \hat{f} \triangleright 1. \tag{9}$$

Realization $\varphi$ that corresponds to the symmetrization map $\Omega_\varphi^{-1} \colon \mathcal{X} \to \hat{\mathcal{X}}$

$$\Omega_\varphi^{-1}(x_{i_1} x_{i_2} \ldots x_{i_\mu}) = \frac{1}{k!} \sum_{\sigma \in S_k} \hat{x}_{i_{\sigma(1)}} \hat{x}_{i_{\sigma(2)}} \ldots \hat{x}_{i_{\sigma(\mu)}} \tag{10}$$

is called the Weyl symmetric realization of $\mathfrak{g}$.

This property is equivalent with

$$\Big(\sum_{\mu=1}^{n} k_\mu \hat{X}_\mu\Big)^m \triangleright 1 = \Big(\sum_{\mu=1}^{n} k_\mu x_\mu\Big)^m, \ \ \forall k_\mu \in \mathbb{K} \text{ and } m \geq 1. \tag{11}$$

Let $\mathbf{C} = [\mathbf{C}_{\mu\nu}]$ be the $n \times n$ matrix of differential operators

$$\mathbf{C}_{\mu\nu} = \sum_{\alpha=1}^{n} C_{\mu\alpha\nu}\partial_\alpha, \tag{12}$$

where $C_{\mu\alpha\nu}$ are the structure constants of the Lie algebra (1).
Define the matrix of formal power series of differential operators

$$e^{\mathbf{C}} = \sum_{m=0}^{\infty} \frac{1}{m!}\mathbf{C}^m. \tag{13}$$

## Theorem

*Let $\psi(t)$ denote the generating function of the Bernoulli numbers $B_k$,*

$$\psi(t) = \frac{t}{1 - e^{-t}} = \sum_{k=0}^{\infty} B_k \frac{(-1)^k}{k!} t^k. \tag{14}$$

*Then the symmetric realization of $X_\mu$ is given by*

$$\hat{X}_\mu = \sum_{\alpha=1}^{n} x_\alpha \psi(\mathbf{C})_{\mu\alpha} = \sum_{\alpha=1}^{n} x_\alpha \left(\frac{\mathbf{C}}{1 - e^{-\mathbf{C}}}\right)_{\mu\alpha}. \tag{15}$$

# Star product

## Definition

The star product $*\colon X \otimes X \to X$ induced by a realization $\varphi\colon \mathfrak{g} \to \hat{A}_n$ is defined with

$$f * g = \Omega_\varphi(\Omega_\varphi^{-1}(f)\Omega_\varphi^{-1}(g)), \quad f, g \in X. \tag{16}$$

The star product is deformation of ordinary commutative product in $X$ since

$$\lim_{h\to 0} f * g = fg. \tag{17}$$

## Proposition

1. $(X, +, *)$ is a unital associative algebra.
2. $\Omega_\varphi\colon \hat{X} \to X_*$ is isomorphism of algebras,

$$\Omega_\varphi(\hat{f}\hat{g}) = \Omega_\varphi(\hat{f}) * \Omega_\varphi(\hat{g}), \ \ \forall \hat{f}, \hat{g} \in X. \tag{18}$$

The structure of the algebra $X_*$ is determined by the relations

$$[x_\mu, x_\nu]_* = \sum_{\alpha=1}^{n} C_{\mu\nu\alpha} x_\alpha. \tag{19}$$

# Realization of the orthogonal algebra $so(n)$

Let $so(n)$ be orthogonal algebra with standard basis $\{M_{\mu\nu} \mid 1 \leq \mu < \nu \leq n\}$ satisfying the commutation relations

$$[M_{\mu\nu}, M_{\lambda\rho}] = \delta_{\nu\lambda} M_{\mu\rho} - \delta_{\mu\lambda} M_{\nu\rho} - \delta_{\nu\rho} M_{\mu\lambda} + \delta_{\mu\rho} M_{\nu\lambda}, \tag{20}$$

where $M_{\mu\nu} = -M_{\nu\mu}$. The commutation relations for $so(n)$ can be written as

$$[M_{\mu\nu}, M_{\lambda\rho}] = \sum_{\alpha,\beta=1}^{n} C_{(\mu\nu)(\lambda\rho)(\alpha\beta)} M_{\alpha\beta}, \tag{21}$$

where structure constants are given by

$$C_{(\mu\nu)(\lambda\rho)(\alpha\beta)} = \delta_{\mu\alpha}\delta_{\rho\beta}\delta_{\nu\lambda} - \delta_{\nu\alpha}\delta_{\rho\beta}\delta_{\mu\lambda} + \delta_{\lambda\alpha}\delta_{\mu\beta}\delta_{\nu\rho} - \delta_{\lambda\alpha}\delta_{\nu\beta}\delta_{\mu\rho}. \tag{22}$$

Generalized Heisenberg algebra $\mathcal{H}_n$ is unital, associative algebra generated by $x_{\mu\nu}, \partial_{\mu\nu}, \ 1 \leq \mu < \nu \leq n$, satisfying $x_{\mu\nu} = -x_{\nu\mu}, \ \partial_{\mu\nu} = -\partial_{\nu\mu}$ and commutation relations

$$\left[x_{\mu\nu}, x_{\alpha\beta}\right] = 0, \quad \left[\partial_{\mu\nu}, \partial_{\alpha\beta}\right] = 0 \quad \text{and} \quad \left[\partial_{\mu\nu}, x_{\alpha\beta}\right] = \delta_{\mu\alpha}\delta_{\nu\beta} - \delta_{\mu\beta}\delta_{\nu\alpha}. \tag{23}$$

Let

$$M_a = \frac{1}{2}\sum_{\mu\nu=1}^{n} \Gamma_a^{\mu\nu} M_{\mu\nu}, \quad a = 1, \ldots, N, \tag{24}$$

where the coefficients $\Gamma_a^{\mu\nu} = -\Gamma_a^{\nu\mu}$.

The inverse transformation is given by

$$M_{\mu\nu} = \sum_{a=1}^{N} \Gamma^a_{\mu\nu} M_a, \tag{25}$$

where the coefficients $\Gamma^a_{\mu\nu} = -\Gamma^a_{\nu\mu}$. and $N = \frac{n(n-1)}{2}$.

We require that $M_\alpha$ generate a Lie algebra defined by commutation relations

$$[M_a, M_b] = \sum_c C_{abc} M_c, \tag{26}$$

with structure constants

$$C_{abc} = \frac{1}{2} \sum_{\alpha,\beta,\lambda=1}^{n} \left( \Gamma_a^{\alpha\lambda} \Gamma_b^{\lambda\beta} - \Gamma_b^{\alpha\lambda} \Gamma_a^{\lambda\beta} \right) \Gamma_{\alpha\beta}^c. \tag{27}$$

Let us define a transformation $(x_{\mu\nu}, \partial_{\mu\nu}) \mapsto (x_\alpha, \partial_\alpha)$ by

$$x_a = \frac{1}{2} \sum_{\mu\nu=1}^{n} \Gamma_a^{\mu\nu} x_{\mu\nu}, \quad \partial_a = \frac{1}{2} \sum_{\mu\nu=1}^{n} \Gamma_a^{\mu\nu} \partial_{\mu\nu}, \ 1 \le a \le N. \tag{28}$$

The inverse transformation is given by

$$x_{\mu\nu} = \sum_{a=1}^{N} \Gamma_{\mu\nu}^a x_a, \quad \partial_{\mu\nu} = \sum_{a=1}^{N} \Gamma_{\mu\nu}^a \partial_a, \quad 1 \le \mu, \nu \le n. \tag{29}$$

We find that

$$[x_a, x_b] = [\partial_a, \partial_b] = 0, \ [\partial_a, x_b] = \delta_{ab}. \tag{30}$$

We can use (15) to write symmetric realization of $M_a$,

$$\hat{M}_a = \sum_{b=1}^{N} x_b \psi(\mathbf{C})_{ab}, \tag{31}$$

where $\mathbf{C}_{ab} = \sum_{c=1}^{N} C_{acb}\partial_c$ and $C_{acb}$ are the structure constants given by (27). It follows from (27) i (29) that

$$\mathbf{C}_{ab} = \frac{1}{4} \sum_{\theta,\sigma=1}^{n} \sum_{\mu\nu=1}^{n} \sum_{\alpha\beta=1}^{n} \Gamma_a^{\mu\nu} \Gamma_{\alpha\beta}^{b} C_{(\mu\nu)(\theta\sigma)(\alpha\beta)} \partial_{\theta\sigma}. \tag{32}$$

If we denote

$$\mathbf{K}_{(\mu\nu)(\alpha\beta)} = \frac{1}{2} \sum_{\theta,\sigma=1}^{n} C_{(\mu\nu)(\theta\sigma)(\alpha\beta)} \partial_{\theta\sigma}, \tag{33}$$

then

$$\mathbf{C}_{ab} = \frac{1}{2} \sum_{\mu\nu=1}^{n} \sum_{\alpha\beta=1}^{n} \Gamma_a^{\mu\nu} \Gamma_{\alpha\beta}^{b} \mathbf{K}_{(\mu\nu)(\alpha\beta)}. \tag{34}$$

Let $\mathbf{K} = \left[\mathbf{K}_{(\mu\nu)(\alpha\beta)}\right]$ and let $\psi(t)$ denote the generating function of the Bernoulli numbers, then

$$\psi(\mathbf{K})_{(\mu\nu)(\alpha\beta)} = \sum_{m=0}^{\infty} \frac{(-1)^m}{m!} (\mathbf{K}^m)_{(\mu\nu)(\alpha\beta)}\,, \tag{35}$$

where powers of $\mathbf{K}$ are given recursively by

$$(\mathbf{K}^m)_{(\mu\nu)(\alpha\beta)} = \sum_{\theta\sigma=1}^{n} \mathbf{K}_{(\mu\nu)(\theta\sigma)} \left(\mathbf{K}^{m-1}\right)_{(\theta\sigma)(\alpha\beta)}\,,\ m \geq 1 \tag{36}$$

and by definition

$$\left(\mathbf{K}^0\right)_{(\mu\nu)(\alpha\beta)} = \frac{1}{2}\left(\delta_{\mu\alpha}\delta_{\nu\beta} - \delta_{\mu\beta}\delta_{\nu\alpha}\right). \tag{37}$$

We find that powers of $\mathbf{K}$ are polynomials in $\partial_{\mu\nu}$ explicity given by

$$\mathbf{K}^m_{(\mu\nu)(\alpha\beta)} = \frac{1}{2}\sum_{k=0}^{m} \binom{m}{k}(-\partial_{\mu\beta}^{m-k}\partial_{\nu\alpha}^{k} + \partial_{\mu\alpha}^{k}\partial_{\nu\beta}^{n-k}),\ m \geq 0. \tag{38}$$

We proved that

$$\psi(\mathbf{C})_{ab} = \frac{1}{2} \sum_{\mu\nu\alpha\beta=1}^{n} \Gamma_a^{\mu\nu} \Gamma_{\alpha\beta}^{b} \psi(\mathbf{K})_{(\mu\nu)(\alpha\beta)}. \tag{39}$$

## Theorem

Realization of the rotation generators $M_{\mu\nu}$ by formal power series in the generalized Heisenbeg algebra $\mathcal{H}_n$ is given by

$$\hat{M}_{\mu\nu} = \sum_{\alpha\beta=1}^{n} x_{\alpha\beta} \psi(\mathbf{K})_{(\mu\nu)(\alpha\beta)}. \tag{40}$$

# Realization of the Lorentz algebra $so(1, n-1)$

Lorentz algebra $so(1, n-1)$ is defined by

$$[M_{\mu\nu}, M_{\lambda\rho}] = \eta_{\nu\lambda} M_{\mu\rho} - \eta_{\mu\lambda} M_{\nu\rho} - \eta_{\nu\rho} M_{\mu\lambda} + \eta_{\mu\rho} M_{\nu\lambda}, \tag{41}$$

where $\eta = \mathrm{diag}(-1, 1, \dots, 1)$ is the Minkowski metric.

The generalized Heisenberg algebra here denoted $\tilde{\mathcal{H}}_n$, is defined by commutation relations

$$[x_{\mu\nu}, x_{\alpha\beta}] = 0, \quad [\partial_{\mu\nu}, \partial_{\alpha\beta}] = 0 \quad \text{and} \quad [\partial_{\mu\nu}, x_{\alpha\beta}] = \eta_{\mu\alpha}\eta_{\nu\beta} - \eta_{\mu\beta}\eta_{\nu\alpha}. \tag{42}$$

The elements of the matrix $\mathbf{K} = \left[\mathbf{K}_{(\mu\nu)(\alpha\beta)}\right]$ are defined by

$$K_{(\mu\nu)(\alpha\beta)} = \frac{1}{2}\left(\eta_{\mu\alpha}\partial_{\nu\beta} - \eta_{\nu\beta}\partial_{\nu\alpha} + \eta_{\nu\beta}\partial_{\nu\alpha} - \eta_{\nu\alpha}\partial_{\mu\beta}\right) \tag{43}$$

and the powers of $\mathbf{K}$ are given recursively by

$$(\mathbf{K}^m)_{(\mu\nu)(\alpha\beta)} = \sum_{\lambda\rho=1}^{n}\sum_{\lambda'\rho'=1}^{n} \mathbf{K}_{(\mu\nu)(\lambda\rho)}\eta_{\lambda\lambda'}\eta_{\rho\rho'}\left(\mathbf{K}^{m-1}\right)_{(\lambda'\rho')(\alpha\beta)}, \tag{44}$$

where

$$\left(\mathbf{K}^0\right)_{(\mu\nu)(\alpha\beta)} = \frac{1}{2}\left(\eta_{\mu\alpha}\eta_{\nu\beta} - \eta_{\mu\beta}\eta_{\nu\alpha}\right). \tag{45}$$

Then the Lorentz algebra (41) admits the realization

$$\hat{M}_{\mu\nu} = \sum_{\alpha\beta=1}^{n}\sum_{\alpha'\beta'=1}^{n} x_{\alpha\beta}\eta_{\alpha\alpha'}\eta_{\beta\beta'}\psi(\mathbf{K})_{(\mu\nu)(\alpha'\beta')}. \tag{46}$$

# Realization of Poincaré algebra

Classical Poincaré algebra $\mathcal{P}(P_\mu, M_{\mu\nu})$ is defined by

$$[P_\mu, P_\nu] = 0, \tag{47}$$

$$[M_{\mu\nu}, P_\lambda] = \eta_{\nu\lambda} P_\mu - \eta_{\nu\lambda} P_\nu \tag{48}$$

$$[M_{\mu\nu}, M_{\rho\sigma}] = \eta_{\nu\rho} M_{\mu\sigma} - \eta_{\mu\rho} M_{\nu\sigma} - \eta_{\nu\sigma} M_{\mu\rho} + \eta_{\mu\sigma} M_{\nu\rho}, 1 \leq \mu\nu \leq n. \tag{49}$$

A realization of the algebra (47) − (113) can be constructed as follows.

First we extend the algebra $\tilde{\mathcal{H}}_n$ by adding to relations (42) pairs of generators $(p_\mu, \partial_\mu),\ 1 \leq \mu \leq n$ such that

$$[p_\mu, p_\nu] = [\partial_\mu, \partial_\nu] = 0,\ [\partial_\mu, p_\nu] = \eta_{\mu\nu}. \tag{50}$$

with all cross-commutators being zero.

Let $\tilde{\mathbf{K}}$ be an upper-trangular matrix

$$\tilde{\mathbf{K}} = \begin{bmatrix} A & 0 \\ C & D \end{bmatrix} \tag{51}$$

with blocks $A$, $C$ and $D$ defined by

$$A_{\mu\nu} = \partial_{\mu\nu} \tag{52}$$

$$C_{(\mu\nu)\alpha} = \eta_{\alpha\mu}\partial_\nu - \eta_{\alpha\nu}\partial_\mu, \tag{53}$$

$$D_{(\mu\nu)(\alpha\beta)} = \frac{1}{2}\left(\eta_{\mu\alpha}\eta_{\nu\beta} - \eta_{\mu\beta}\eta_{\nu\alpha}\right). \tag{54}$$

Then the Poincaré algebra $\mathcal{P}$ admits the realization

$$\hat{P}_\mu = \sum_{\alpha=1}^{n}\sum_{\alpha'=1}^{n} p_\alpha \eta_{\alpha\alpha'}\psi(\tilde{\mathbf{K}})_{\mu\alpha'}, \tag{55}$$

$$\hat{M}_{\mu\nu} = \sum_{\alpha\beta=1}^{n}\sum_{\alpha'\beta'=1}^{n} x_{\alpha\beta}\eta_{\alpha\alpha'}\eta_{\beta\beta'}\psi(\tilde{\mathbf{K}})_{(\mu\nu)(\alpha'\beta')} + \sum_{\alpha=1}^{n}\sum_{\alpha'=1}^{n} p_\alpha\eta_{\alpha\alpha'}\psi(\tilde{\mathbf{K}})_{(\mu\nu)\alpha'}. \tag{56}$$

# Yang algebra

Let us start with the Yang algebra

$$[\hat{x}_\mu, \hat{x}_\nu] = \frac{i\epsilon_1}{M^2} M_{\mu\nu}, \quad [\hat{p}_\mu, \hat{p}_\nu] = \frac{i\epsilon_2}{R^2} M_{\mu\nu}, \tag{57}$$

$$[M_{\mu\nu}, \hat{x}_\lambda] = i\left(\eta_{\mu\lambda}\hat{x}_\nu - \eta_{\nu\lambda}\hat{x}_\mu\right), \quad [M_{\mu\nu}, \hat{p}_\lambda] = i\left(\eta_{\mu\lambda}\hat{p}_\nu - \eta_{\nu\lambda}\hat{p}_\mu\right), \tag{58}$$

$$[\hat{x}_\mu, \hat{p}_\nu] = i\eta_{\mu\nu}\hat{h}, \quad \left[\hat{h}, \hat{x}_\mu\right] = \frac{i\epsilon_1}{M^2}\hat{p}_\mu, \quad \left[\hat{h}, \hat{p}_\mu\right] = -\frac{i\epsilon_2}{R^2}\hat{x}_\mu, \tag{59}$$

$$\left[M_{\mu\nu}, \hat{h}\right] = 0, \tag{60}$$

$$[M_{\mu\nu}, M_{\rho\sigma}] = i\left(\eta_{\mu\rho}M_{\nu\sigma} - \eta_{\mu\sigma}M_{\nu\rho} - \eta_{\nu\rho}M_{\mu\sigma} + \eta_{\nu\sigma}M_{\mu\rho}\right), \tag{61}$$

where $\epsilon_1^2 = \epsilon_2^2 = 1$ and $\eta_{\mu\nu} = diag(-1, 1, 1, 1)$.

Let us consider the case $\epsilon_1 = \epsilon_2 = 1$ corresponding to the $o(1,5)$ algebra.
Let us define the most general new generators linear in $\hat{x}_\mu$, $\hat{p}_\mu$ and $M_{\mu\nu}$ introducing parameters $\kappa$ and $\tilde{\kappa}$,

$$\tilde{X}_\mu = u\left(\cos\varphi \hat{x}_\mu + \frac{R}{M}\sin\varphi \hat{p}_\mu\right) + \frac{1}{\kappa} a_\rho M_{\mu\rho}, \tag{62}$$

$$\tilde{P}_\mu = v\left(\cos\psi \hat{p}_\mu + \frac{M}{R}\sin\psi \hat{x}_\mu\right) + \frac{1}{\tilde{\kappa}} b_\rho M_{\mu\rho}, \tag{63}$$

with the Lorentz generators $\tilde{M}_{\mu\nu} = M_{\mu\nu}$ unchanged. The parameters $u, v, \varphi, \psi$ and four-vectors $a_\mu, b_\mu$ are dimensionless with $uv \neq 0$.
The inverse transformations are

$$\hat{x}_\mu = \frac{u^{-1}\frac{1}{R}\cos\psi\left(\tilde{X}_\mu - \frac{1}{\kappa}a_\rho M_{\mu\rho}\right) - v^{-1}\frac{1}{M}\sin\varphi\left(\tilde{P}_\mu - \frac{1}{\tilde{\kappa}}b_\rho M_{\mu\rho}\right)}{\frac{1}{R}\cos(\varphi+\psi)}, \tag{64}$$

$$\hat{p}_\mu = \frac{v^{-1}\frac{1}{M}\cos\varphi\left(\tilde{P}_\mu - \frac{1}{\tilde{\kappa}}b_\rho M_{\mu\rho}\right) - u^{-1}\frac{1}{R}\sin\psi\left(\tilde{X}_\mu - \frac{1}{\kappa}a_\rho M_{\mu\rho}\right)}{\frac{1}{M}\cos(\varphi+\psi)}. \tag{65}$$

The generators $\tilde{X}_\mu$, $\tilde{P}_\mu$, $M_{\mu\nu}$ and $\tilde{H}$ generate a new class of Lie algebras isomorphic to the initial Yang algebra. These algebras are defined by

$$\left[\tilde{X}_\mu, \tilde{X}_\nu\right] = i\left(\frac{1}{M^2}\left(u^2 + a^2\frac{M^2}{\kappa^2}\right)M_{\mu\nu} + \frac{1}{\kappa}\left(a_\mu\tilde{X}_\nu - a_\nu\tilde{X}_\mu\right)\right), \tag{66}$$

$$\left[\tilde{P}_\mu, \tilde{P}_\nu\right] = i\left(\frac{1}{R^2}\left(v^2 + b^2\frac{R^2}{\tilde{\kappa}^2}\right)M_{\mu\nu} + \frac{1}{\tilde{\kappa}}\left(b_\mu\tilde{P}_\nu - b_\nu\tilde{P}_\mu\right)\right), \tag{67}$$

$$\left[\tilde{X}_\mu, \tilde{P}_\nu\right] = i\left(\eta_{\mu\nu}\tilde{H} + \frac{1}{\tilde{\kappa}}b_\mu\tilde{X}_\nu - \frac{1}{\kappa}a_\nu\tilde{P}_\mu + \frac{1}{MR}\left(uv\sin(\varphi+\psi) + abMR\frac{1}{\kappa\tilde{\kappa}}\right)M_{\mu\nu}\right), \tag{68}$$

$$\left[M_{\mu\nu}, \tilde{X}_\lambda\right] = i\left(\eta_{\mu\lambda}\tilde{X}_\nu - \eta_{\nu\lambda}\tilde{X}_\mu + \frac{1}{\kappa}\left(a_\mu M_{\lambda\nu} - a_\nu M_{\lambda\mu}\right)\right), \tag{69}$$

$$\left[M_{\mu\nu}, \tilde{P}_\lambda\right] = i\left(\eta_{\mu\lambda}\tilde{P}_\nu - \eta_{\nu\lambda}\tilde{P}_\mu + \frac{1}{\tilde{\kappa}}\left(b_\mu M_{\lambda\nu} - b_\nu M_{\lambda\mu}\right)\right), \tag{70}$$

where

$$\tilde{H} = \hat{h}uv\cos(\varphi+\psi) + \frac{1}{\kappa}a\tilde{P} - \frac{1}{\tilde{\kappa}}b\tilde{X} - \frac{1}{\kappa\tilde{\kappa}}a_\rho b_\sigma M_{\rho\sigma} \tag{71}$$

and

$$\left[M_{\mu\nu}, \tilde{H}\right] = i\left(\frac{1}{\tilde{\kappa}}\left(b_\nu\tilde{X}_\mu - b_\mu\tilde{X}_\nu\right) - \frac{1}{\kappa}\left(a_\nu\tilde{P}_\mu - a_\mu\tilde{P}_\nu\right)\right), \tag{72}$$

$$\left[\tilde{H}, \tilde{X}_\mu\right] = i\left(\frac{1}{M^2}\left(u^2 + a^2\frac{M^2}{\kappa^2}\right)\tilde{P}_\mu - \frac{1}{MR}\left(uv\sin(\varphi+\psi) + abMR\frac{1}{\kappa\tilde{\kappa}}\right)\tilde{X}_\mu - \frac{1}{\kappa}a_\mu\tilde{H}\right), \quad (73)$$

$$\left[\tilde{H}, \tilde{P}_\mu\right] = -i\left(\frac{1}{R^2}\left(v^2 + b^2\frac{R^2}{\tilde{\kappa}^2}\right)\tilde{X}_\mu - \frac{1}{MR}\left(uv\sin(\varphi+\psi) + abMR\frac{1}{\kappa\tilde{\kappa}}\right)\tilde{P}_\mu + \frac{1}{\tilde{\kappa}}b_\mu\tilde{H}\right). \quad (74)$$

If we define generators

$$M_{\mu 4} = M\hat{x}_\mu, \; M_{\mu 5} = R\hat{p}_\mu, \; M_{45} = MR\hat{h} \quad (75)$$

then the Yang algebra (57)- (61) takes the form of an orthogonal algebra $o(1,5)$ with

$$[M_{AB}, M_{CD}] = i\left(\eta_{AC}M_{BD} - \eta_{AD}M_{BC} + \eta_{BD}M_{AC} - \eta_{BC}M_{AD}\right), \quad (76)$$

where $A, B, C, D = 0,1,2,3,4,5, \; \eta_{AB} = \mathrm{diag}(-1,1,1,1,1,1)$.

If we define

$$\tilde{M}_{\mu 4} = M\tilde{X}_{\mu},\ \tilde{M}_{\mu 5} = R\tilde{P}_{\mu},\ \tilde{M}_{45} = MR\tilde{H} \tag{77}$$

then the algebra (66)-(74) becomes the $o(1,5,g)$ algebra with

$$\left[\tilde{M}_{AB}, \tilde{M}_{CD}\right] = i\left(g_{AC}\tilde{M}_{BD} - g_{AD}\tilde{M}_{BC} + g_{BD}\tilde{M}_{AC} - g_{BC}\tilde{M}_{AD}\right), \tag{78}$$

where the metric $g_{AB}$ is given by a symmetric matrix defined as

$$\begin{aligned} &g_{\mu\nu} = \eta_{\mu\nu},\ g_{\mu 4} = \frac{M}{\kappa}a_{\mu},\ g_{\mu 5} = \frac{R}{\tilde{\kappa}}b_{\mu},\ g_{44} = u^2 + a^2\frac{M^2}{\kappa^2},\ g_{55} = v^2 + b^2\frac{R^2}{\tilde{\kappa}^2}, \\ &g_{45} = g_{54} = uv\sin(\varphi + \psi) + ab\frac{MR}{\kappa\tilde{\kappa}}. \end{aligned} \tag{79}$$

It follows that

$$\det g = (g_{45} - g_{\mu 4}g_{\mu 5})^2 - (g_{44} - g_{\mu 4}g_{\mu 4})(g_{55} - g_{\nu 5}g_{\nu 5}) < 0, \tag{80}$$

i.e. using above formulae for $g_{AB}$ (79) we get $\det g = -u^2v^2\cos^2(\varphi + \psi)$.

We have that $g_{AB} = \left(S\eta S^T\right)_{AB}$, $\det S^2 = -\det g$ with

$$S = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ -\frac{M}{\kappa}a_0 & \frac{M}{\kappa}a_1 & \frac{M}{\kappa}a_2 & \frac{M}{\kappa}a_3 & \sigma & 0 \\ -\frac{R}{\tilde{\kappa}}b_0 & \frac{R}{\tilde{\kappa}}b_1 & \frac{R}{\tilde{\kappa}}b_2 & \frac{R}{\tilde{\kappa}}b_3 & \upsilon & \tau \end{bmatrix}, \tag{81}$$

where $\sigma = u$, $\upsilon = v\sin(\varphi + \psi)$ and $\tau = v\cos(\varphi + \psi)$.

The relations between $\tilde{M}_{AB}$ and $M_{AB}$ can be written as $\tilde{M}_{AB} = \left(SMS^T\right)_{AB}$ with same matrix $S$ as above.

If we impose $g_{44} = 0$ and $g_{55} = 0$, we obtain

$$a^2 = -u^2 \frac{\kappa^2}{M^2} < 0, \quad b^2 = -v^2 \frac{\tilde{\kappa}^2}{R^2} < 0 \tag{82}$$

and then it follows

$$\left[\tilde{X}_\mu, \tilde{X}_\nu\right] = \frac{i}{\kappa}\left(a_\mu \tilde{X}_\nu - a_\nu \tilde{X}_\mu\right) \tag{83}$$

and

$$\left[\tilde{P}_\mu, \tilde{P}_\nu\right] = \frac{i}{\tilde{\kappa}}\left(b_\mu \tilde{P}_\nu - b_\nu \tilde{P}_\mu\right). \tag{84}$$

The relation $\left[\tilde{X}_\mu, \tilde{X}_\nu\right]$ defines $\kappa$-Minkowski space and $\left[\tilde{P}_\mu, \tilde{P}_\nu\right]$ describes the Born dual $\tilde{\kappa}$-Minkowski space.

In particular, in the limit when $R \to \infty$ and $\frac{b_\mu}{\tilde{\kappa}} = 0$ we get the $\kappa$-Minkowski space with the $\kappa$-Poincaré algebra, with $a^2 \leq 0$ and

$$\left[\tilde{X}_\mu, \tilde{X}_\nu\right] = \frac{i}{\kappa}\left(a_\mu \tilde{X}_\nu - a_\nu \tilde{X}_\mu\right), \tag{85}$$

$$\left[\tilde{P}_\mu, \tilde{P}_\nu\right] = 0 \tag{86}$$

$$\left[\tilde{X}_\mu, \tilde{P}_\nu\right] = i\left(\eta_{\mu\nu}\tilde{H} - \frac{1}{\kappa}a_\nu \tilde{P}_\mu\right), \tag{87}$$

$$\left[M_{\mu\nu}, \tilde{X}_\lambda\right] = i\left(\eta_{\mu\lambda}\tilde{X}_\nu - \eta_{\nu\lambda}\tilde{X}_\mu + \frac{1}{\kappa}\left(a_\mu M_{\lambda\nu} - a_\nu M_{\lambda\mu}\right)\right), \tag{88}$$

$$\left[M_{\mu\nu}, \tilde{P}_\lambda\right] = i\left(\eta_{\mu\lambda}\tilde{P}_\nu - \eta_{\nu\lambda}\tilde{P}_\mu\right), \tag{89}$$

$$\left[M_{\mu\nu}, \tilde{H}\right] = \frac{-i}{\kappa}\left(a_\nu \tilde{P}_\mu - a_\mu \tilde{P}_\nu\right), \tag{90}$$

$$\left[\tilde{H}, \tilde{X}_\mu\right] = \frac{-i}{\kappa}a_\mu \tilde{H}, \tag{91}$$

$$\left[\tilde{H}, \tilde{P}_\mu\right] = 0. \tag{92}$$

The algebras $o(1,5,g)$, $o(2,4,g)$ and $o(3,3,g)$ generated with the $\tilde{M}_{AB}$ are defined in

$$\left[\tilde{M}_{AB}, \tilde{M}_{CD}\right] = i\tilde{C}_{AB,CD}{}^{EF}\tilde{M}_{EF} = i\left(g_{AC}\tilde{M}_{BD} - g_{AD}\tilde{M}_{BC} + g_{BD}\tilde{M}_{AC} - g_{BC}\tilde{M}_{AD}\right), \qquad (93)$$

with $g_{AB} = \left(S\eta S^T\right)_{AB}$ corresponding to the $o(1,5,g)$, $o(2,4,g)$, $o(3,3,g)$.
Starting with the generalized Heisenberg algebra defined with the commutative coordinates $X_{AB}$ and their corresponding momenta $K^{AB}$

$$[X_{AB}, X_{CD}] = \left[K^{AB}, K^{CD}\right] = 0, \quad \left[X_{AB}, K^{CD}\right] = i\left(\delta_A{}^C\delta_B{}^D - \delta_A{}^D\delta_B{}^C\right), \qquad (94)$$

where $K^{CD} = g^{CM}g^{DN}K_{MN}$.

Using the general Weyl realization of the Lie algebras corresponding to the Weyl symmetric ordering we express the generators $\tilde{M}_{AB}$ with

$$\tilde{M}_{AB} = X_{CD} \left( \frac{\tilde{\mathbf{C}}}{1 - e^{-\tilde{\mathbf{C}}}} \right)_{AB}^{\quad CD}, \tag{95}$$

where

$$\tilde{\mathbf{C}}_{AB}{}^{CD} = -\frac{1}{2} \tilde{C}_{AB,EF}{}^{CD} K^{EF} \tag{96}$$

and

$$\tilde{C}_{AB,EF}{}^{CD} = \frac{1}{2} \left[ -g_{BE} \left( \delta_A{}^C \delta_F{}^D - \delta_A{}^D \delta_F{}^C \right) + g_{AF} \left( \delta_E{}^C \delta_B{}^D - \delta_E{}^D \delta_B{}^C \right) - (A \leftrightarrow B) \right]. \tag{97}$$

and we find up to first order

$$\tilde{M}_{AB} = X_{AB} + \frac{1}{2} \left( X_{AE} K_B{}^E - X_{BE} K_A{}^E \right) \tag{98}$$

and

$$\left[ \tilde{M}_{AB}, K^{CD} \right] = i \left( \delta_A{}^C \delta_B{}^D - \delta_A{}^D \delta_B{}^C \right) + \frac{i}{2} \left( \delta_A{}^C K_B{}^D - \delta_B{}^C K_A{}^D + \delta_B{}^D K_A{}^C - \delta_A{}^D K_B{}^C \right). \tag{99}$$

The Weyl realization $\tilde{M}^{AB}$ enjoys property

$$e^{\frac{1}{2}t^{AB}\tilde{M}^{AB}} \triangleright 1 = e^{\frac{1}{2}t^{AB}X^{AB}}, \tag{100}$$

where $t_{AB}$ are real numbers transforming as tensors, and action $\triangleright$ is defined with

$$X_{AB} \triangleright f(X_{EF}) = X_{AB}f(X_{EF}), \quad K^{AB} \triangleright f(X_{EF}) = -i\frac{\partial f(X_{EF})}{\partial X_{AB}} = \left[K^{AB}, f(X_{EF})\right]. \tag{101}$$

In particular

$$X_{AB} \triangleright 1 = X_{AB}, \; K_{AB} \triangleright 1 = 0, \tag{102}$$

$$K^{AB} \triangleright e^{\frac{1}{2}t^{EF}X^{EF}} = t^{AB}e^{\frac{1}{2}t^{EF}X^{EF}}. \tag{103}$$

We can write the generators $X_{AB}$ and $K^{AB}$ in the terms of the four dimensional variables

$$K^{\mu 4} = \frac{Q^{\mu}}{M}, \quad K^{\mu 5} = \frac{Y^{\mu}}{R}, \quad K^{45} = \frac{W}{MR}, \tag{104}$$

$$X_{\mu 4} = MX_{\mu}, \quad X_{\mu 5} = RP_{\mu}, \quad X_{45} = MRH, \tag{105}$$

so that

$$[X_{\mu}, Q_{\nu}] = i\delta_{\mu}^{\nu}, \quad [P_{\mu}, Y_{\nu}] = i\delta_{\mu}^{\nu}, \quad [H, W] = i. \tag{106}$$

The generators $\tilde{M}_{AB}$ can be written in the terms of the four dimensional variables with greek indices ($\tilde{M}_{\mu\nu},\ \tilde{X}_\mu,\ \tilde{P}_\mu,\ \tilde{H}$ expressed in the terms of $M_{\mu\nu},\ X_\mu,\ P_\mu,\ H,$ and $K_{\mu\nu},\ Q_\mu,\ Y_\mu,\ W$). They are

$$\begin{aligned}\tilde{M}_{\mu\nu} = X_{\mu\nu} + \frac{1}{2}\Big(X_{\mu\alpha}\left(K_\nu{}^\alpha - \frac{1}{M}g_{\nu 4}Q^\alpha - \frac{1}{R}g_{\nu 5}Y^\alpha\right) \\ + X_\mu\left(Q_\nu - \frac{1}{R}g_{\nu 5}W\right) + P_\mu\left(Y_\nu + \frac{1}{M}g_{\nu 4}W\right) - (u \leftrightarrow v)\Big),\end{aligned} \tag{107}$$

$$\begin{aligned}\tilde{X}_\mu = X_\mu + \frac{1}{2}\Big(-\frac{1}{M}X_{\mu\nu}\left(g_{\alpha 4}K^{\nu\alpha} + \frac{1}{M}g_{44}Q^\nu + \frac{1}{R}g_{45}Y^\nu\right) + \frac{1}{M}X_\mu\left(g_{\nu 4}Q^\nu - \frac{1}{R}g_{45}W\right) \\ + X_\nu\left(K_\mu{}^\nu - \frac{1}{M}g_{\mu 4}Q^\nu - \frac{1}{R}g_{\mu 5}Y^\nu\right) + \frac{1}{M}P_\mu\left(g_{\nu 4}Y^\nu + \frac{1}{M}g_{44}W\right) - H\left(Y_\mu + \frac{1}{M}g_{\mu 4}W\right)\Big),\end{aligned} \tag{108}$$

$$\begin{aligned}\tilde{P}_\mu = P_\mu + \frac{1}{2}\Big(-\frac{1}{R}X_{\mu\nu}\left(g_{\alpha 5}K^{\nu\alpha} + \frac{1}{M}g_{45}Q^\nu + \frac{1}{R}g_{55}Y^\nu\right) + \frac{1}{R}P_\mu\left(g_{\nu 5}Y^\nu + \frac{1}{M}g_{45}W\right) \\ + P_\nu\left(K_\mu{}^\nu - \frac{1}{R}g_{\mu 5}Y^\nu - \frac{1}{M}g_{\mu 4}Q^\nu\right) + \frac{1}{R}X_\mu\left(g_{\nu 5}Q^\nu - \frac{1}{R}g_{55}W\right) + H\left(Q_\mu - \frac{1}{R}g_{\mu 5}W\right)\Big),\end{aligned} \tag{109}$$

$$
\begin{aligned}
\tilde{H} = H + \frac{1}{2}\Big(\frac{1}{R}X_{\mu}\left(g_{\nu 5}K^{\mu\nu} + \frac{1}{M}g_{45}Q^{\mu} + \frac{1}{R}g_{55}Y^{\mu}\right) \\
-\frac{1}{M}P_{\mu}\left(\frac{1}{M}g_{44}Q^{\mu} + \frac{1}{R}g_{45}Y^{\mu} + g_{\nu 4}K^{\mu\nu}\right) + H\left(\frac{1}{R}g_{\mu 5}Y^{\mu} + \frac{1}{M}g_{\mu 4}Q^{\mu}\right)\Big).
\end{aligned} \tag{110}
$$

For the dual $\kappa$-Minkowski spaces $g_{44} = 0, \; g_{55} = 0$.
The other realizations can be obtain from the Weyl realizations of $\tilde{M}^{W}_{AB}$ by using similarity transformation

$$
\tilde{M}_{AB} = \mathcal{S}\tilde{M}^{W}_{AB}\mathcal{S}^{-1}, \tag{111}
$$

where $\mathcal{S} = \exp(G)$, with $G = XF(K)$. The corresponding realizations will be linear in $X$ and can be written as a series in $K$.

Defining

$$\exp\left(\frac{i}{2}s^{AB}\tilde{M}_{AB}\right)\exp\left(\frac{i}{2}t^{CD}\tilde{M}_{CD}\right)=\exp\left(\frac{i}{2}\left(s^{AB}\oplus t^{AB}\right)\tilde{M}_{AB}\right)=\exp\left(\frac{i}{2}\mathcal{D}^{AB}(s,t)\tilde{M}_{AB}\right), \tag{112}$$

where $s^{AB}$ and $t^{AB}$ transform as tensors of corresponding orthogonal algebra. One has at first order

$$\mathcal{D}^{AB}\left(s^{CD},t^{CD}\right)=s^{AB}+t^{AB}-\frac{1}{2}\left(s^{AC}t^{B}{}_{C}-s^{BC}t^{A}{}_{C}\right). \tag{113}$$

In the following all formulae are written in the first order. The coproduct $\Delta K^{AB}$ is

$$\Delta K^{AB}=\mathcal{D}^{AB}\left(K^{AB}\otimes 1,1\otimes K^{AB}\right)=\Delta_0 K^{AB}-\frac{1}{2}\left(K^{AC}\otimes K^{B}{}_{C}-K^{BC}\otimes K^{A}{}_{C}\right), \tag{114}$$

where $\Delta_0 K^{AB}=K^{AB}\otimes 1+1\otimes K^{AB}$.

In components, it reads

$$\Delta K^{\mu\nu} = \Delta_0 K^{\mu\nu} - \frac{1}{2}\Big( K^{\mu\alpha}\otimes K^{\nu}{}_{\alpha} + \frac{1}{M^2} g_{44} Q^{\mu}\otimes Q^{\nu} + \frac{1}{R^2} g_{55} Y^{\mu}\otimes Y^{\nu}$$
$$+ \frac{1}{MR} g_{45}\left(Q^{\mu}\otimes Y^{\nu} + Y^{\mu}\otimes Q^{\nu}\right) + \frac{1}{M} g_{\alpha 4}\left(K^{\mu\alpha}\otimes Q^{\nu} + Q^{\mu}\otimes K^{\nu\alpha}\right)$$
$$+ \frac{1}{R} g_{\alpha 5}\left(K^{\mu\alpha}\otimes Y^{\nu} + Y^{\mu}\otimes K^{\nu\alpha}\right) - (u \leftrightarrow v)\Big), \tag{115}$$

$$\Delta Q^{\mu} = \Delta_0 Q^{\mu} - \frac{1}{2}\Big( - K^{\mu\alpha}\otimes Q_{\alpha} + Q_{\alpha}\otimes K^{\mu\alpha} + \frac{1}{MR} g_{45}\left(Q^{\mu}\otimes W - W\otimes Q^{\mu}\right)$$
$$+ \frac{1}{R^2} g_{55}\left(Y^{\mu}\otimes W - W\otimes Y^{\mu}\right) + \frac{1}{R} g_{\alpha 5}\left(K^{\mu\alpha}\otimes W - W\otimes K^{\mu\alpha}\right)$$
$$- \frac{1}{M} g_{\alpha 4}\left(Q^{\mu}\otimes Q^{\alpha} - Q^{\alpha}\otimes Q^{\mu}\right) - \frac{1}{R} g_{\alpha 5}\left(Y^{\mu}\otimes Q^{\alpha} - Q^{\alpha}\otimes Y^{\mu}\right)\Big), \tag{116}$$

$$\Delta Y^{\mu} = \Delta_0 Y^{\mu} - \frac{1}{2}\Big( - K^{\mu\alpha}\otimes Y_{\alpha} + Y_{\alpha}\otimes K^{\mu\alpha} - \frac{1}{MR} g_{45}\left(Y^{\mu}\otimes W - W\otimes Y^{\mu}\right)$$
$$+ \frac{1}{M^2} g_{44}\left(W\otimes Q^{\mu} - Q^{\mu}\otimes W\right) - \frac{1}{M} g_{\alpha 4}\left(K^{\mu\alpha}\otimes W - W\otimes K^{\mu\alpha}\right)$$
$$- \frac{1}{M} g_{\alpha 4}\left(Q^{\mu}\otimes Y^{\alpha} - Y^{\alpha}\otimes Q^{\mu}\right) - \frac{1}{R} g_{\alpha 5}\left(Y^{\mu}\otimes Y^{\alpha} - Y^{\alpha}\otimes Y^{\mu}\right)\Big), \tag{117}$$

$$\Delta W = \Delta_0 W - \frac{1}{2}\Big( Q^{\alpha}\otimes Y_{\alpha} - Y^{\alpha}\otimes Q_{\alpha} + \frac{1}{R} g_{\alpha 5}\left(Y^{\alpha}\otimes W - W\otimes Y^{\alpha}\right)$$
$$\frac{1}{M} g_{\alpha 5}\left(Q^{\alpha}\otimes W - W\otimes Q^{\alpha}\right)\Big). \tag{118}$$

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