Defects in four-dimensional continua: a paradigm for the expansion of the universe?

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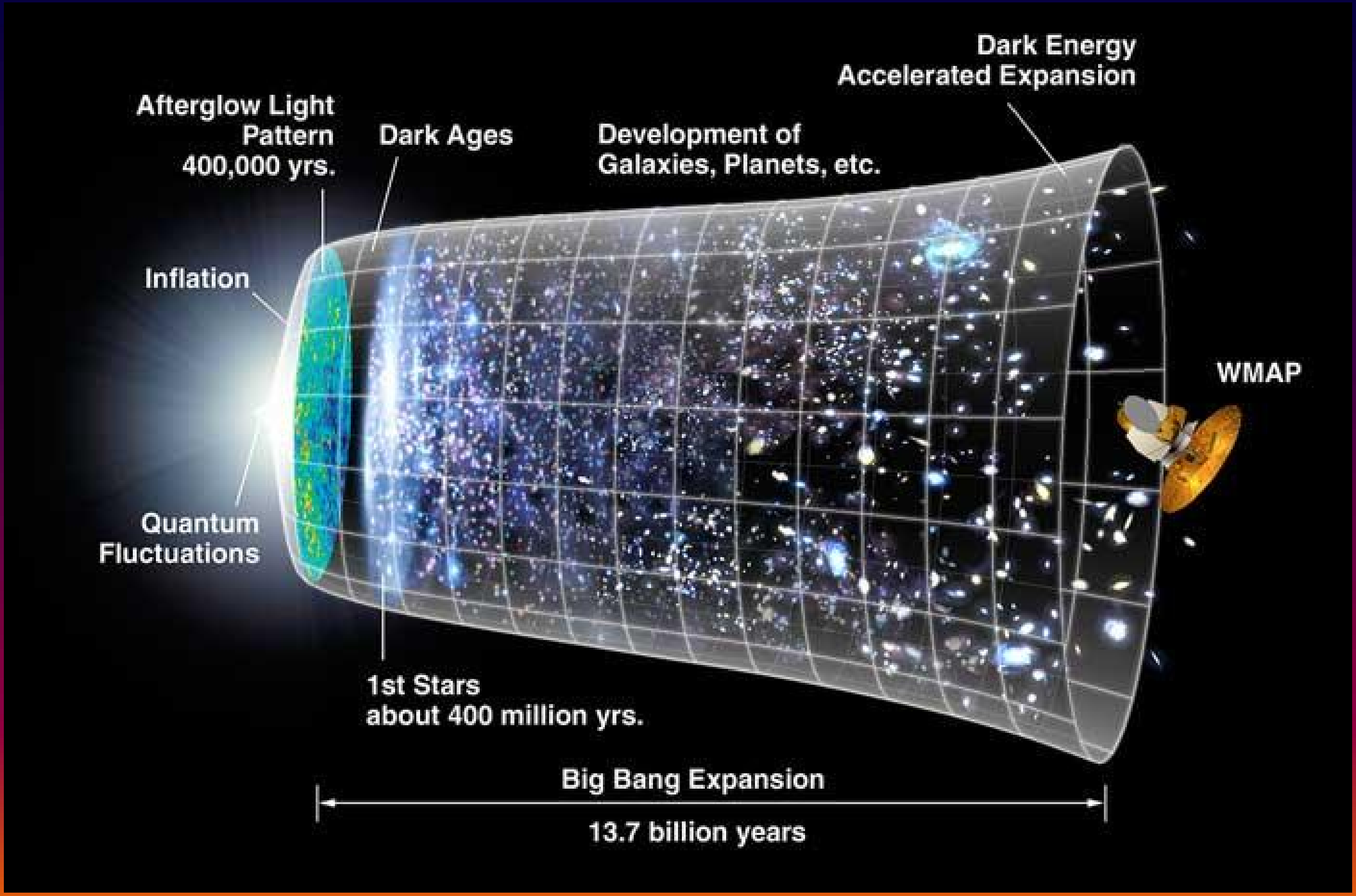
Plan of the talk

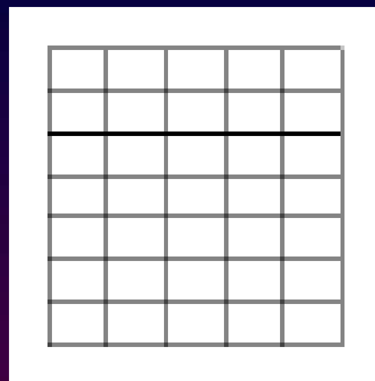
- Physical motivation
- Elasticity in N dimensions
- Defects and curvature
- Extension to space-time
- A "pre-shaped" space time: the Cosmic Defect theory
- Defects and vector theories

Presently agreed evolution of the universe



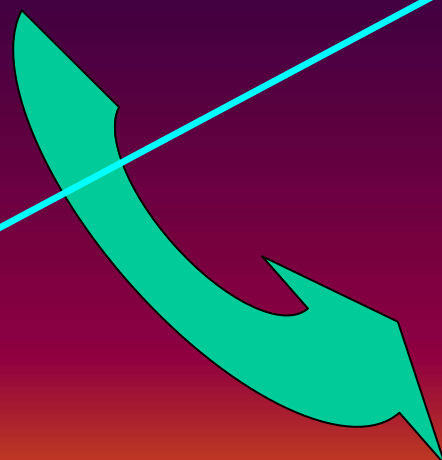
Elasticity

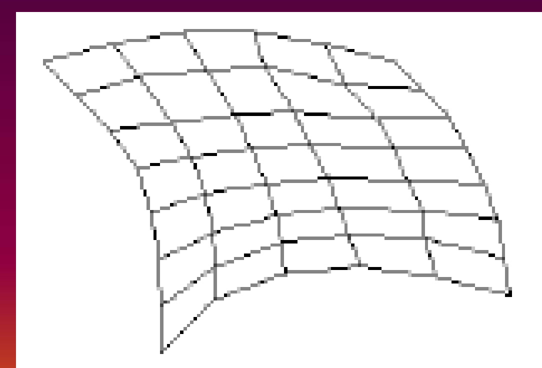
"Natural" manyfold



$$dx^\mu = \phi^\mu{}_a d\xi^a$$

Actual manyfold





$$dx^\mu = \frac{\partial x^\mu}{\partial \xi^a} d\xi^a$$

Geometry and elasticity

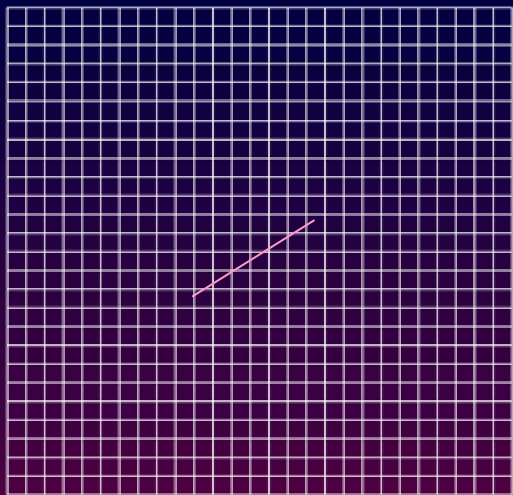
In a strained medium each point is in one to one correspondence with points in the unstrained state

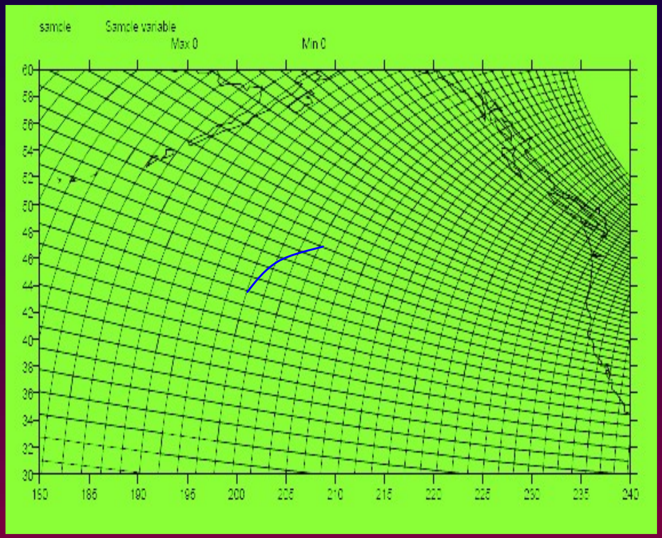
$$x^{\mu} = \xi^{\mu} + u^{\mu}$$

The new situation is diffeomorphic to the old one

u is a function of x

Lengths





$$dl^2 = \eta_{ab} d\xi^a d\xi^b$$

$$dl'^2 = g_{\mu\nu} dx^\mu dx^\nu$$

The strain tensor

$$dl'^2 - dl^2 = \left(g_{\mu\nu} - \eta_{ab}\phi^a{}_\mu \phi^b{}_\nu\right) dx^\mu dx^\nu = 2\varepsilon_{\mu\nu} dx^\mu dx^\nu$$

$$\varepsilon_{\mu\nu} = \frac{1}{2}\left(g_{\mu\nu} - \eta_{ab}\phi^a{}_\mu \phi^b{}_\nu\right) = \frac{1}{2}\left(\eta_{a\mu}\phi^a{}_\nu + \eta_{b\nu}\phi^b{}_\mu + \eta_{ab}\phi^a{}_\mu \phi^b{}_\nu\right)$$

$$\varepsilon_{\mu\nu} = \frac{1}{2}\left(\frac{\partial u_\mu}{\partial x^\nu} + \frac{\partial u_\nu}{\partial x^\mu} + \frac{\partial u_a}{\partial x^\nu}\frac{\partial u^a}{\partial x^\mu}\right)$$

Strain tensor

Integrability

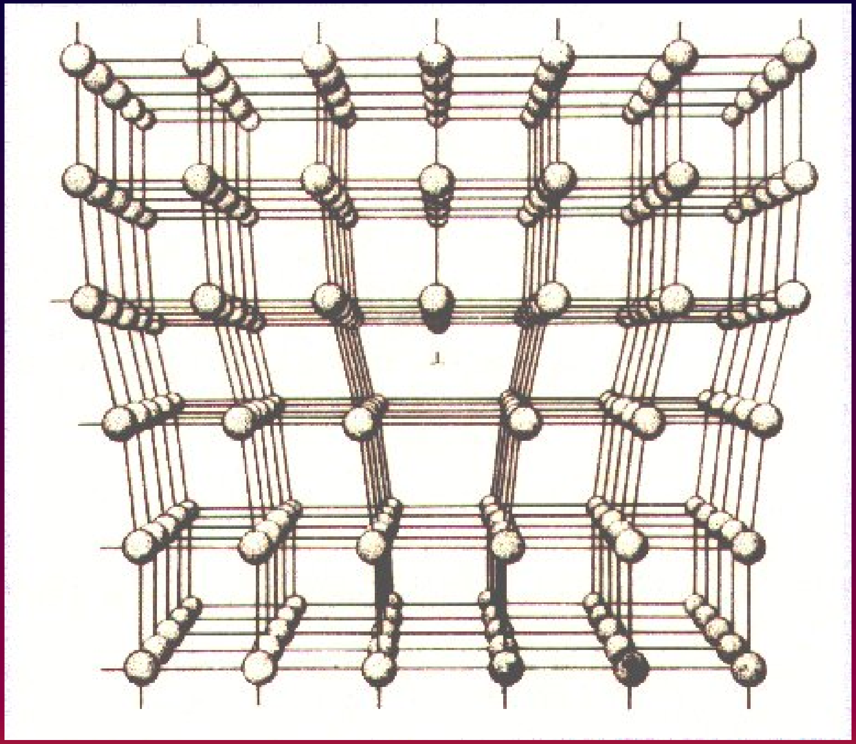
$$g_{\mu\nu} = \eta_{ab} \frac{\partial \xi^a}{\partial x^\mu} \frac{\partial \xi^b}{\partial x^\nu}$$

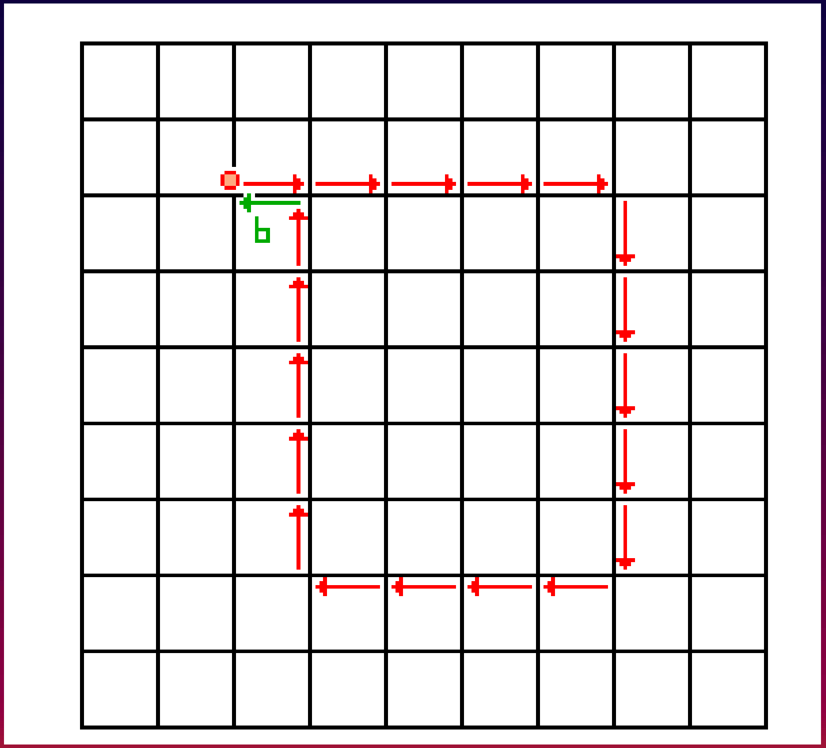
De Saint Venant integrability condition

$$R^\alpha{}_{\beta\mu\nu} = 0$$

Pure elasticity

Defects





A dislocation

Burgers vector

$$b^{\mu} = -\oint \phi^{\mu}{}_{\alpha} dx^{\alpha} \neq 0$$

Non-integrable form

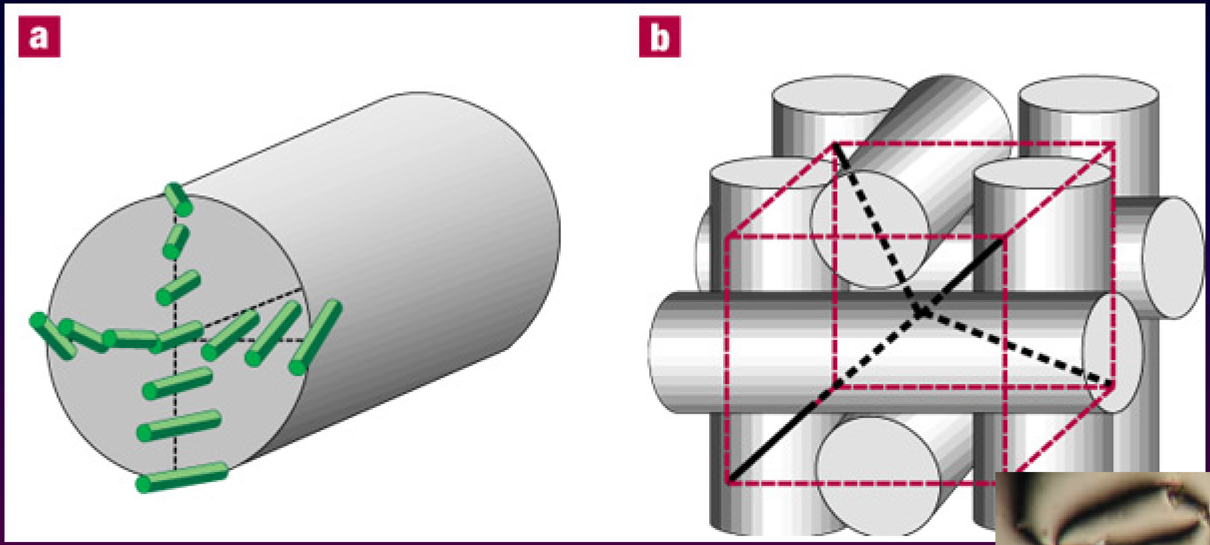
$$b^{\mu} = -\oint T^{\mu}{}_{\alpha\beta} dx^{\beta} \wedge dx^{\alpha} \neq 0$$

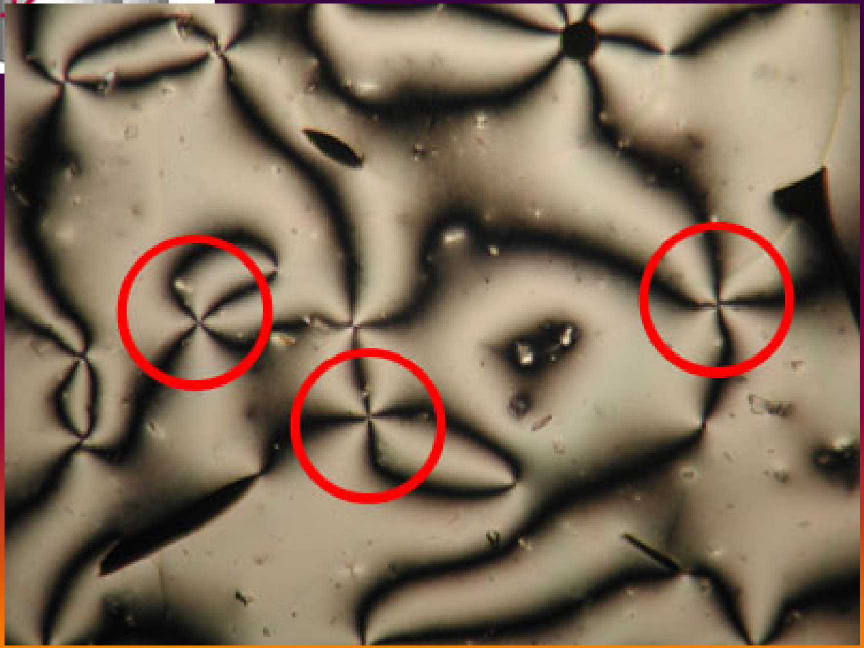
Dislocation density

$$T^{\mu}{}_{\alpha\beta} = \frac{\partial \Phi^{\mu}{}_{\alpha}}{\partial x^{\beta}} - \frac{\partial \Phi^{\mu}{}_{\beta}}{\partial x^{\alpha}}$$

Torsion

Disclinations





$$\Delta v^{\mu} = -\oint R^{\mu}{}_{\nu\alpha\beta} v^{\nu} dx^{\alpha} \wedge dx^{\beta}$$

General deformation

$$r' = \Lambda(r)\cdot r + T(r)$$

Local Lorentz transformation

Local translation

Poincaré group

$$r = \left(x^0, x^1, x^2, x^3\right)$$

Defects: the soldering one-form

$$\omega = \Gamma^T + dx + \Gamma^L x$$

Translation connection

Lorentz connection

$$ds^2 = \eta_{\mu\nu}\omega^\mu \otimes \omega^\nu$$

10 types of independent edge topological defects

Many other kinds of defects

Hooke's law

$$\sigma^{\mu\nu} = C^{\mu\nu}{}_{\alpha\beta}\,\varepsilon^{\alpha\beta}$$

Elastic modulus tensor

Isotropic medium

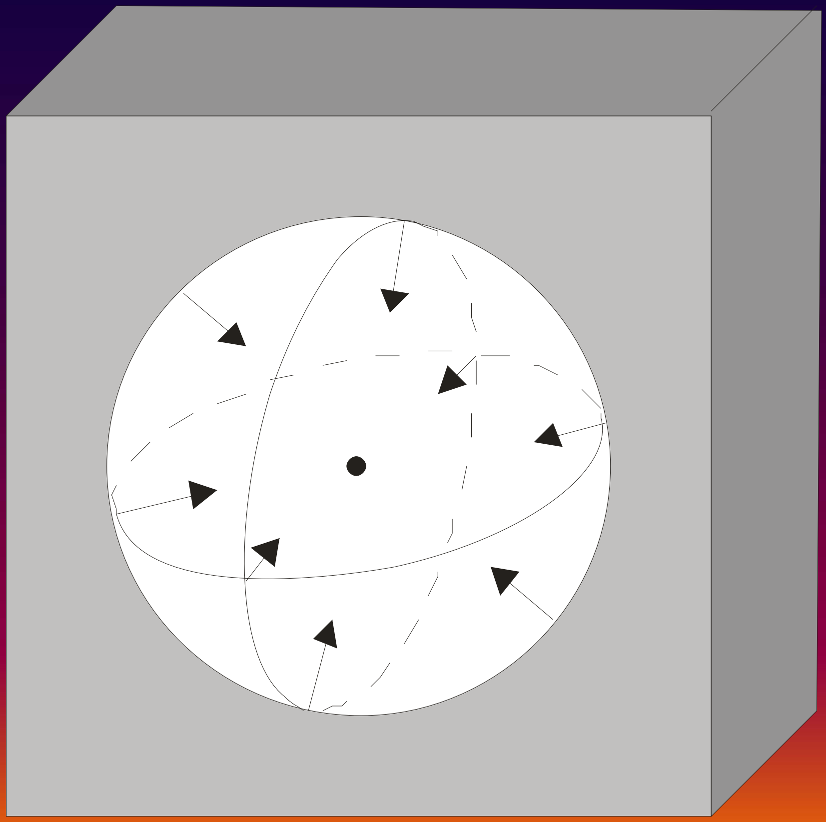
$$C_{\alpha\beta\mu\nu} = \lambda\eta_{\alpha\beta}\eta_{\mu\nu} + \mu\left(\eta_{\alpha\mu}\eta_{\beta\nu} + \eta_{\alpha\nu}\eta_{\beta\mu}\right)$$

Lamé coefficients

$$\sigma^{\mu\nu} = \lambda\eta^{\mu\nu}\varepsilon + 2\mu\varepsilon^{\mu\nu}$$

Lorentz signature notation

Defects: an interesting example



"Radial" displacement field

$$\Psi = \left(u(t), 0, 0, 0\right)$$

$$\frac{\partial \psi^0}{\partial t} = \frac{\partial u}{\partial t}; \quad \frac{\partial \psi^0}{\partial r} = u; \quad \frac{\partial \psi^0}{\partial \theta} = ur; \quad \frac{\partial \psi^0}{\partial \varphi} = ur \sin\theta$$

$$\varepsilon_{00} = \frac{1}{2}\left(2\frac{\partial u}{\partial t} + \left(\frac{\partial u}{\partial t}\right)^2\right); \quad \varepsilon_{rr} = \frac{u^2}{2}; \quad \varepsilon_{\theta\theta} = \frac{u^2}{2} r^2; \quad \varepsilon_{\varphi\varphi} = \frac{u^2}{2} r^2 \sin^2\theta$$

The line element

Unperturbed $$ds^2 = dt^2 - \left(dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta d\varphi^2\right)$$

$$ds^2 = \left(1 + \frac{du}{dt}\right)^2 dt^2 - \left(1 - u^2\right)\left(dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta d\varphi^2\right)$$ Strained

$$d\tau = \left(1 + \frac{du}{dt}\right) dt \rightarrow \tau = t + u$$

$$ds^2 = d\tau^2 - a^2(\tau)\left(dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta d\varphi^2\right)$$

A Robertson-Walker universe

$$ds^2 = dt^2 - a^2\left(dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta d\varphi^2\right)$$

$$a^2(\tau) = 1 - u^2(\tau)$$

How can we choose a Lagrangian expressing the presence of the defect?

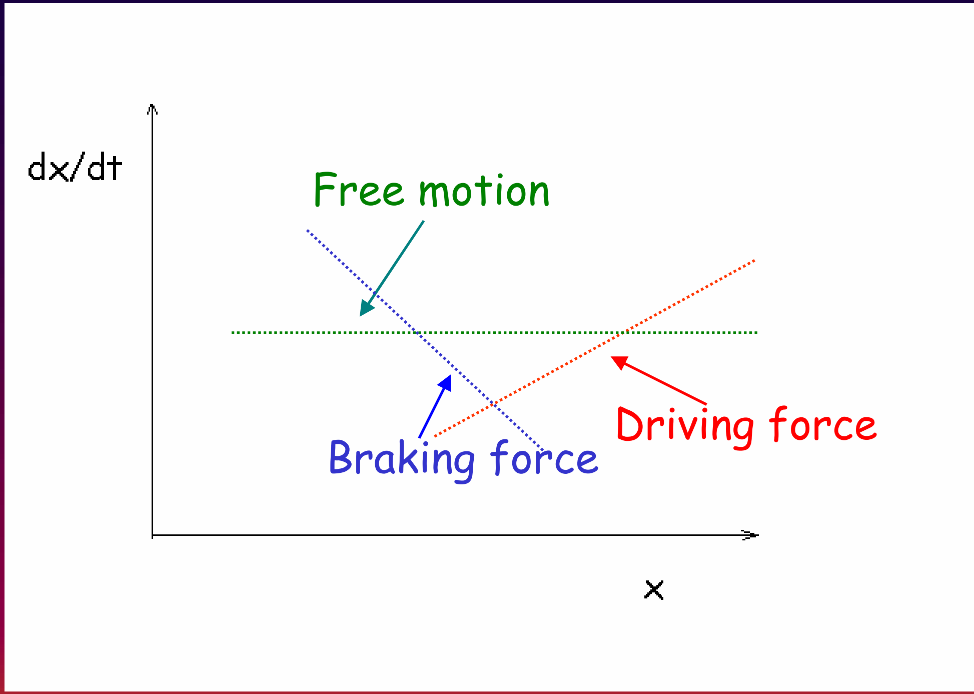
Start from the phase space of a Robertson-Walker universe and look around for similar phase spaces

Phase space analogy

Point particle in an isotropic fluid

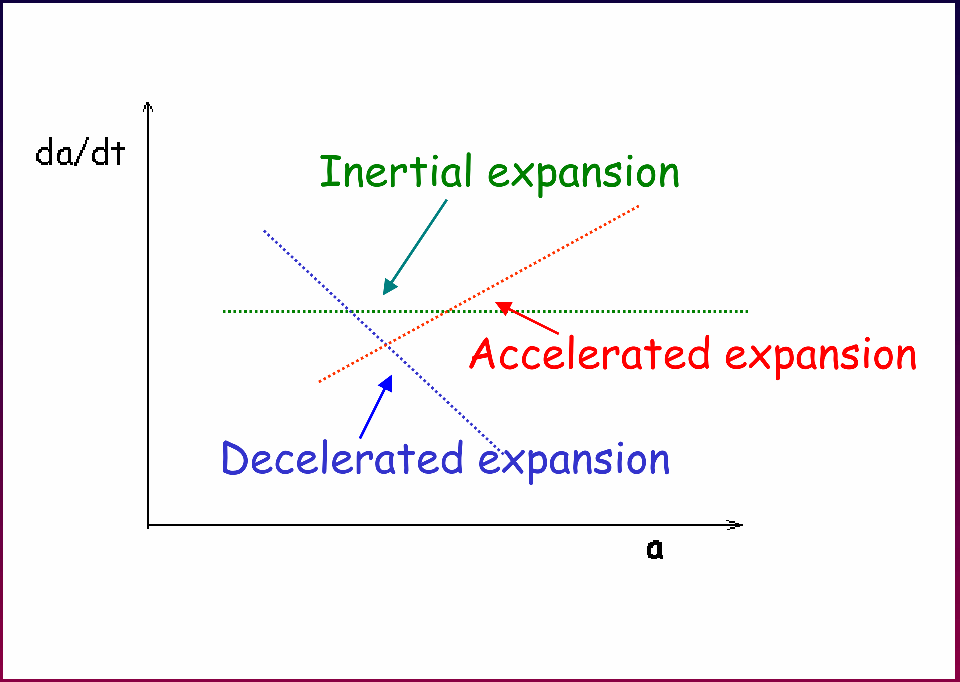
$$S = m\int e^{(\gamma t+\beta x)/m}\dot{x}^2 dt$$

$$S = m\int e^{\eta_{\alpha\beta}r^\alpha\gamma^\beta} ds$$

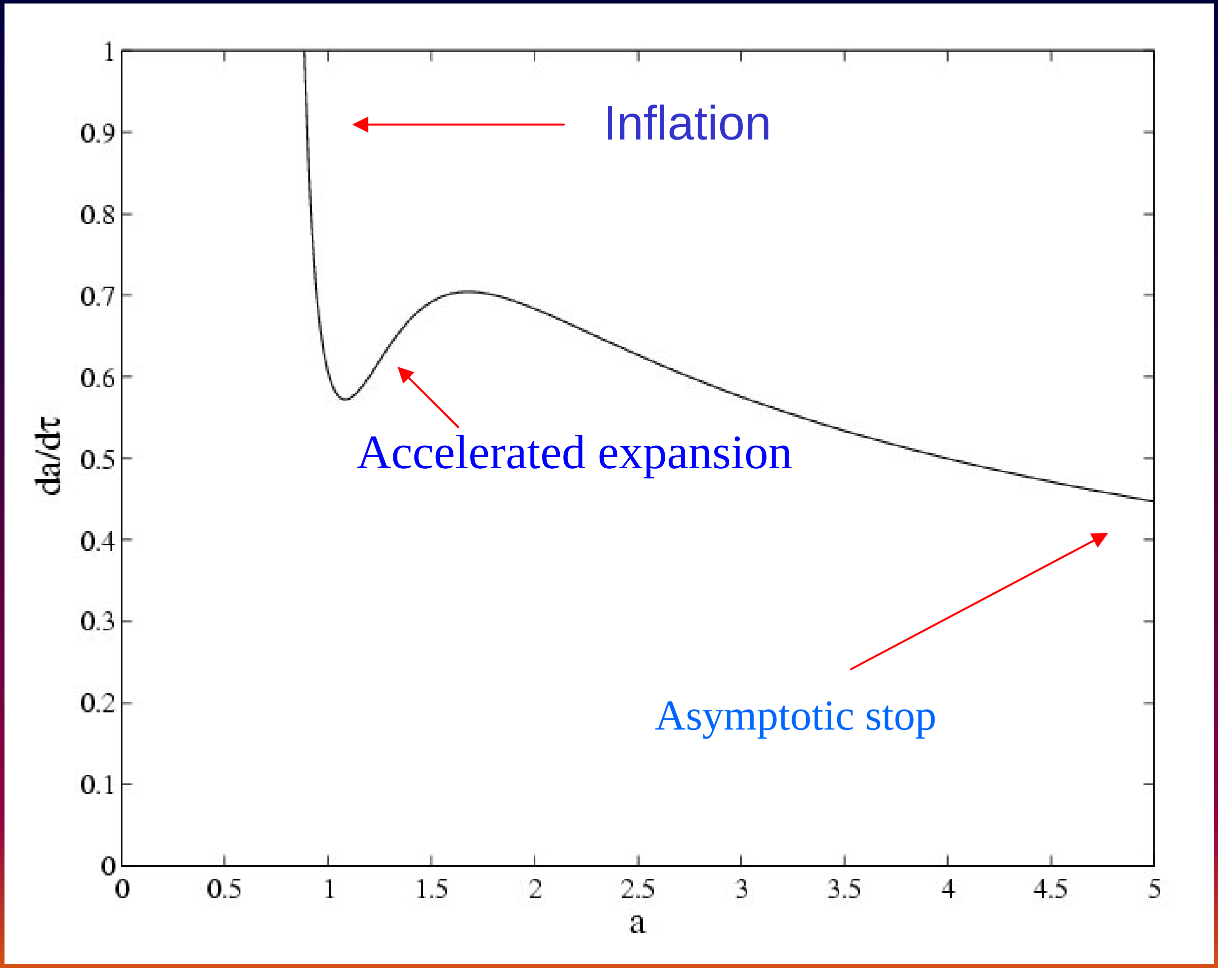


Robertson-Walker universe

$$S = \int e^{-g_{\mu\nu}\gamma^{\mu}\gamma^{\nu}} R\sqrt{-g}\, d^4x$$



Expansion rate



“Elastic” approach

$$S = \int \left(R + \frac{\kappa}{2} \sigma_{\mu\nu} \varepsilon^{\mu\nu} \right) \sqrt{-g}\, d^4 x$$

Isotropy $\sigma_{\mu\nu} \varepsilon^{\mu\nu} = \lambda \varepsilon^2 + 2\mu \varepsilon_{\mu\nu} \varepsilon^{\mu\nu}$

$$S = \int \left(R + \frac{\kappa}{2} \lambda \varepsilon^2 + \kappa \mu \varepsilon_{\mu\nu} \varepsilon^{\mu\nu} \right) \sqrt{-g}\, d^4 x$$

$$\varepsilon = \frac{\dot{u}}{2} \left(2 + \dot{u} \right) - \frac{3}{2} \frac{u^2}{a^2}$$

$$\varepsilon_{\mu\nu} \varepsilon^{\mu\nu} = \frac{\dot{u}^2}{2} \left(2 + \dot{u} \right)^2 + \frac{3}{4} \frac{u^4}{a^4}$$

Fourth order theory

Effective Lagrangian

$$L = 6a + 6a\dot{a}^2 + \frac{\kappa}{2}\left(\frac{\lambda}{4} + \mu\right)a^3\dot{u}^2\left(2+\dot{u}\right)^2$$
$$-\frac{3}{4}\kappa\lambda a u^2\dot{u}\left(2+\dot{u}\right) + \frac{3}{4}\kappa\left(\frac{3}{2}\lambda + \mu\right)\frac{u^4}{a}$$

Field equations

$$12a\ddot{a}-6\dot{a}^2-\frac{3}{2}\kappa\left(\frac{\lambda}{4}+\mu\right)a^2\dot{u}^2\left(2+\dot{u}\right)^2+$$

$$\frac{3}{4}\kappa\lambda u^2\dot{u}\left(2+\dot{u}\right)+\frac{3}{4}\kappa\left(\frac{3}{2}\lambda+\mu\right)\frac{u^4}{a^2}-6=0$$

Vector field equations

$$6\kappa\left(\frac{\lambda}{4}+\mu\right)a^2\dot{a}\dot{u}\left(2+\dot{u}\right)\left(1+\dot{u}\right)+\kappa\left(\frac{\lambda}{4}+\mu\right)a^3\left(2+6\dot{u}+3\dot{u}^2\right)\ddot{u}+$$

$$-\frac{3}{2}\kappa\lambda\dot{a}u^2\left(1+\dot{u}\right)-\frac{3}{2}\kappa\lambda a\left(2u\dot{u}+2u\dot{u}^2+u^2\ddot{u}\right)+$$

$$\frac{3}{2}\kappa\lambda au\dot{u}\left(2+\dot{u}\right)-3\kappa\left(\frac{3}{2}\lambda+\mu\right)\frac{u^3}{a}=0$$

Gravitational analogy

$$\sigma^{\mu\nu} = \lambda g^{\mu\nu}\varepsilon + 2\mu\varepsilon^{\mu\nu}$$

$$\lambda = -\mu$$

$$\varepsilon_{\mu\nu} - \frac{1}{2} g_{\mu\nu}\varepsilon = \frac{1}{2\mu}\sigma_{\mu\nu}$$

$$\mu \geq -2\lambda; \qquad N = 4$$

Einstein equations

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \frac{8\pi G}{c^4} T_{\mu\nu}$$

(Madsen)

Elastic potential

$$\frac{1}{2}C_{\mu\nu\alpha\beta}\varepsilon^{\mu\nu}\varepsilon^{\alpha\beta}=\frac{1}{2}\lambda\left(\varepsilon_{\alpha}{}^{\alpha}\right)^{2}+\mu\varepsilon_{\mu\nu}\varepsilon^{\mu\nu}$$

$$\varepsilon_{\mu\nu}\Leftrightarrow R_{\mu\nu}$$

$$\Phi=\frac{1}{2}R^{2}-\kappa R_{\mu\nu}R^{\mu\nu}$$

f(R)

$$S=\int\left(\frac{R^{2}}{2}-\kappa R_{\mu\nu}R^{\mu\nu}\right)\sqrt{-g}\,d^{4}x$$

General Lagrangian treatment (non-exponential coupling)

$$e^{\delta^{\alpha\beta}+\gamma^{\alpha}\gamma^{\beta}} R_{\alpha\beta} = R + \gamma^{\alpha}\gamma^{\beta} R_{\alpha\beta} + \ldots$$

$$\gamma^{\beta} R_{\alpha\beta} = \left(\nabla_{\alpha}\nabla_{\sigma} - \nabla_{\sigma}\nabla_{\alpha}\right)\gamma^{\sigma}$$

Non-minimal coupling

$$\frac{L}{\sqrt{|g|}} = R + \lambda\nabla_{\alpha}\gamma^{\beta}\nabla_{\beta}\gamma^{\alpha} + \mu\left(\nabla_{\alpha}\gamma^{\alpha}\right)^{2} + \nu\nabla_{\alpha}\gamma^{\beta}\nabla^{\alpha}\gamma_{\beta} + \gamma^{2}\overbrace{\left(1 + R\right)}$$

The equations for a and χ

$$3\left(2+A\chi^2\right)a\dot{a}^2+6B\chi\dot{\chi}a^2\dot{a}+\left(C\dot{\chi}^2-\chi^2\right)a^3=W$$

$$Ca^2\ddot{\chi}+3Ca\dot{a}\dot{\chi}+\left[3Ba\ddot{a}-a^2+\left(6B-3A\right)\dot{a}^2\right]\chi=0$$

$$A=\lambda+\nu+3\mu\pm 2$$

$$B=\mu\pm 2$$

$$C=\lambda+\mu+\nu$$

$$\gamma=\left(\chi,0,0,0\right)$$

Comments

- A "forest" of theories
- Lagrangian engineering
- Need for leading models or paradigms
- Space-time as physical continuum corresponds to f(R) theories, vector-tensor theories, conformal gravity
- CD lends a good guiding idea for exploring cosmological options

Ether again

.... according to the general theory of relativity space is endowed with physical qualities; in this sense, therefore, there exists an ether. According to the general theory of relativity space without ether is unthinkable; for in such space there not only would be no propagation of light, but also no possibility of existence for standards of space and time (measuring-rods and clocks), nor therefore any space-time intervals in the physical sense. But this ether may not be thought of as endowed with the quality characteristic of ponderable media, as consisting of parts which may be tracked through time. The idea of motion may not be applied to it.

Albert Einstein, Leiden, 192

The end

A. Tartaglia, M. Capone, Int. Jour. Mod. Phys. D, **17, 275-** 299 (2008)

A. Tartaglia, N. Radicella, Phys. Rev. D, 76, 083501 (2007)

A. Tartaglia, Int. Jour. Mod. Phys. A, **20**, 2336-2340 (2005)

A. Tartaglia, M. Capone, V. Cardone, N. Radicella, arXiv:0801.1921, to appear on Int. Jour. Mod. Phys. D

Useful readings

- R. A. Madsen, private communication (1988)
- R. A. Puntigam, H. H. Soleng, Class. Quant. Grav. **14**, 1129 (1997)
- MF. Miri, N. Rivier, Jour. Phys. A **35**, 1727 (2002)
- F. W. Hehl, Y. N. Obukhov, Annales de la Fondation Louis de Broglie (2007)

Some history

"Ether is a very wonderful thing. It may exist only in the imagination of the wise, being invented and endowed with properties to suit their hypotheses; but we cannot do without it. How is energy to be transmitted through space without a medium?"

Oliver Heaviside, Electrical Papers

Ether is superfluous

The introduction of a “luminiferous ether” will prove to be superfluous inasmuch as the view here to be developed will not require an “absolutely stationary space”.....

Albert Einstein, 1905

$$\tau = u + T \pm \sqrt{u^2 + \frac{1}{u^{2/3}}}$$

$\tau \to 0 \qquad u \to A$

$\tau \to \infty \qquad u \to 0$

$$\tau = u + T + \sqrt{u^2 + \frac{1}{u^{2/3}}}$$

$$T = -A - \sqrt{A^2 + \frac{1}{A^{2/3}}}$$

Curvatura

$$\mathrm{R} = 6\frac{1 - a\ddot{a} - \dot{a}^2}{a^2}$$

Densità di energia

$$w_0 = 3\frac{\dot{a}^2 - 1}{a^2}$$

Pressione

$$p = \frac{2a\ddot{a} + \dot{a}^2 - 1}{a^2}$$