Noncommutative extensions of the space-time symmetries beyond supersymmetry

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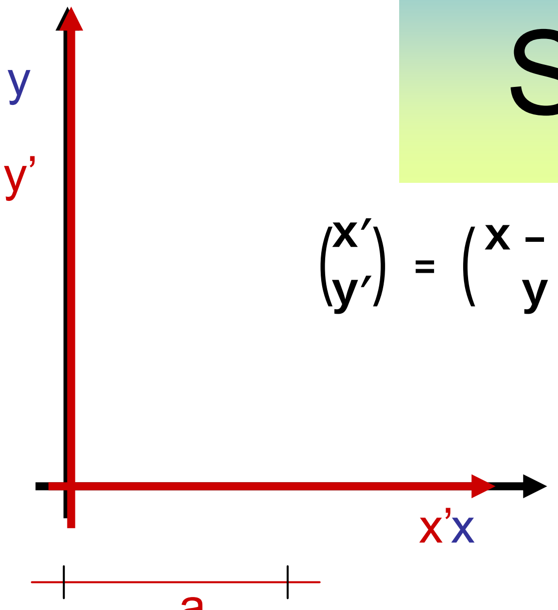
Abstract: Novel bosonic and fermionic graded extensions of the Poincaré algebra beyond supersymmetry are presented. Their nilpotent features and their combination with nonabelian symmetry give the possibility of going beyond Coleman & Mandula no-go theorems.


26.12.2005

Symmetry Generators

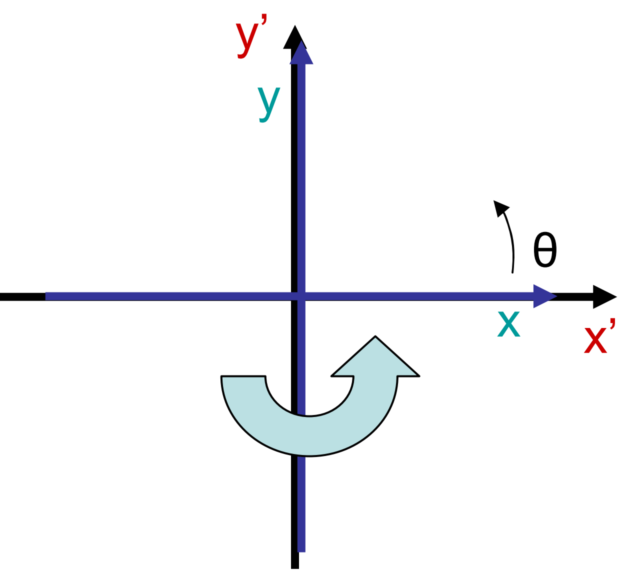


$$\begin{pmatrix}x'\\y'\end{pmatrix} = \begin{pmatrix}x-a\\y\end{pmatrix} = \begin{pmatrix}x\\y\end{pmatrix} - a\begin{pmatrix}1\\0\end{pmatrix} = \begin{pmatrix}x\\y\end{pmatrix} - a\,\partial_x\begin{pmatrix}x\\y\end{pmatrix} = \exp\{-a\,\partial_x\}\begin{pmatrix}x\\y\end{pmatrix}$$

realization of $\{P_x\} = \mathcal{R}_D\,(P_x) = -i\hbar\,\{\partial_x\}$

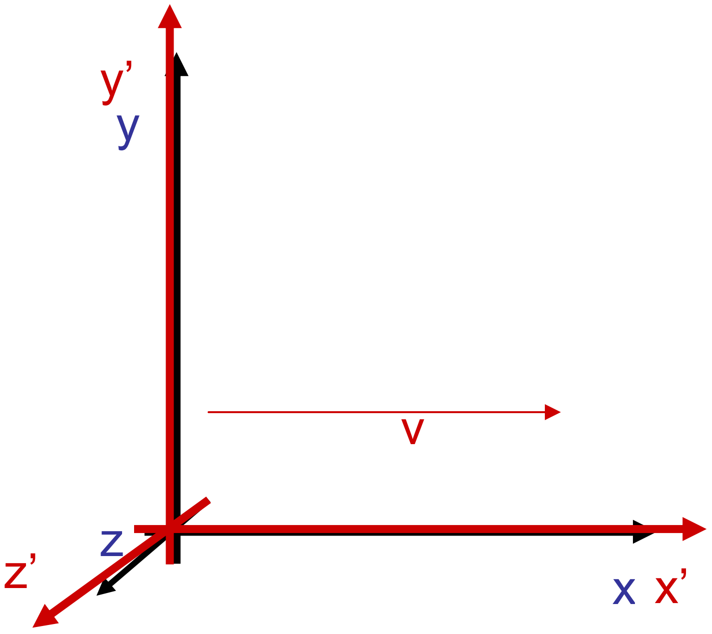
$$\begin{pmatrix}x'\\y'\end{pmatrix} = \exp\left\{\frac{i\,(-a)}{\hbar}\,\mathcal{R}_D\,(P_x)\right\}\begin{pmatrix}x\\y\end{pmatrix}$$

$$\begin{pmatrix}x'\\y'\end{pmatrix} = \begin{pmatrix}\cos(\theta) & -\sin(\theta)\\ \sin(\theta) & \cos(\theta)\end{pmatrix}\begin{pmatrix}x\\y\end{pmatrix} = \exp\left\{\theta\begin{pmatrix}0 & -1\\1 & 0\end{pmatrix}\right\}\begin{pmatrix}x\\y\end{pmatrix}$$



realization of $\{M_{xy}$ or $J_z\} = \mathcal{R}_D\,(M_{xy}) =$
$= \mathcal{R}_D\,(M_{xy}) = -i\hbar\,\{x\,\partial_y - y\,\partial_x\}$

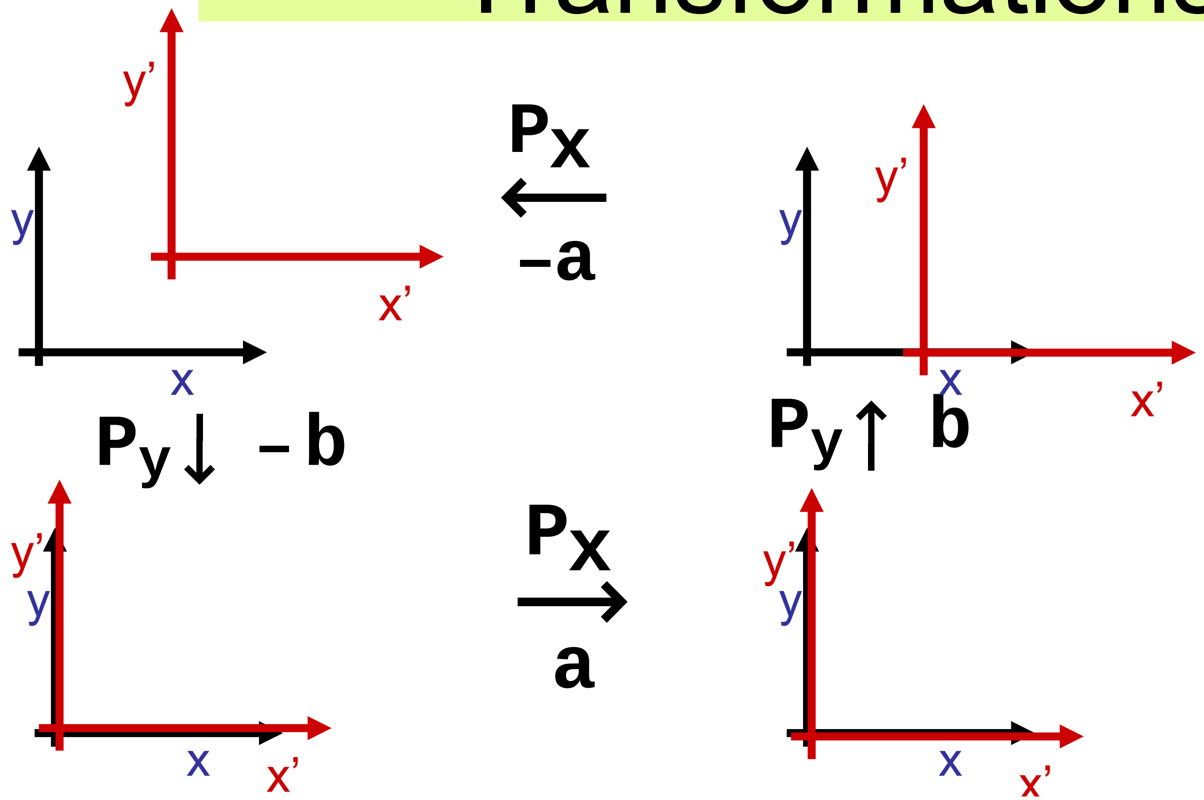
$$\begin{pmatrix}x'\\y'\end{pmatrix} = \exp\left\{\frac{i\theta}{\hbar}\,\mathcal{R}_D\,(M_{xy})\right\}\begin{pmatrix}x\\y\end{pmatrix}$$



$$\begin{pmatrix} ct' \\ x' \\ y' \\ z' \end{pmatrix} = \begin{pmatrix} \frac{ct - vx/c}{\sqrt{1-v^2/c^2}} \\ \frac{x - vt}{\sqrt{1-v^2/c^2}} \\ y \\ z \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{1-v^2/c^2}} & \frac{-v/c}{\sqrt{1-v^2/c^2}} & 0 & 0 \\ \frac{-v/c}{\sqrt{1-v^2/c^2}} & \frac{1}{\sqrt{1-v^2/c^2}} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} ct \\ x \\ y \\ z \end{pmatrix} =$$

$$\exp\left((v/c) \begin{pmatrix} 0 & -1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \right) \begin{pmatrix} ct \\ x \\ y \\ z \end{pmatrix} = \exp\left(\frac{i\,(v/c)}{\hbar} \mathcal{R}_M (M_{01}) \right) \begin{pmatrix} ct \\ x \\ y \\ z \end{pmatrix}$$

Composition of Symmetry Transformations



$$\{1+b\,\partial_y\}\,\{1+a\,\partial_x\}\,\{1-b\,\partial_y\}\,\{1-a\,\partial_x\}+\vartheta\,(a^2\,b^2) = 1+ab\,(\partial_y\partial_x-\partial_x\partial_y)+\vartheta\,(a^2\,b^2) =$$

$$1-\frac{i\,(-a)\,(-b)}{\hbar}\left(\frac{-i}{\hbar}\right)[\mathcal{R}_D\,(P_x)\,,\,\mathcal{R}_D\,(P_y)\,]+\vartheta\,(a^2\,b^2) = 1$$

$$[\mathcal{R}_D\,(P_x)\,,\,\mathcal{R}_D\,(P_y)\,] = 0$$

$[P_x, P_y] = 0$ $[P_x, P_z] = 0$ $[P_y, P_z] = 0$

Underlying and Extending Grading

$$[T_{[0]i}, T_{[0]j}] = \mathrm{i}\,\epsilon_{ijk} T_{[0]k}, \quad [T_{[0]i}, \bar{T}_{[0]j}] = 0, \quad [\bar{T}_{[0]i}, \bar{T}_{[0]j}] = \mathrm{i}\,\epsilon_{ijk} \bar{T}_{[0]k},$$
$$[T_{[0]i}, P_{[0]\nu}] = -\mathrm{i}\,\sigma^P(i,0)_\nu{}^\rho\, P_{[0]\rho}, \quad [\bar{T}_{[0]i}, P_{[0]\nu}] = -\mathrm{i}\, P_{[0]\rho}\, \bar{\sigma}^P(i,0)^\rho{}_\nu$$
$$[P_{[0]\mu}, P_{[0]\nu}] = 0.$$

$$g_{\mu,\nu} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}$$

Index	Generators	Parameters
(0,0) $\equiv [0]0$	$P_0 \equiv P_{[0]0}$	$t \equiv x_{[0]0}$
(1,0) $\equiv [0]1$	$P_1 \equiv P_{[0]1},\ M_{01},\ M_{23}$	$x \equiv x_{[0]1},\ \epsilon_{01},\ \epsilon_{23}$
(0,1) $\equiv [0]2$	$P_2 \equiv P_{[0]2},\ M_{02},\ M_{31}$	$y \equiv x_{[0]2},\ \epsilon_{02},\ \epsilon_{31}$
(1,1) $\equiv [0]3$	$P_3 \equiv P_{[0]3},\ M_{03},\ M_{12}$	$z \equiv x_{[0]3},\ \epsilon_{03},\ \epsilon_{12}$

$$[M_{xy},\ P_x] = i\hbar\, P_y$$

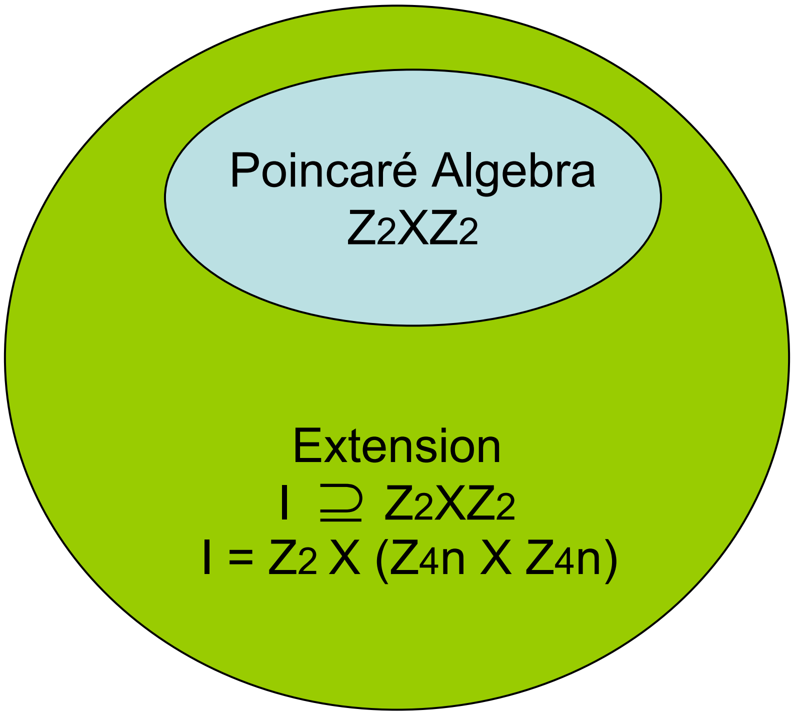
[0]3 +[0]1 =[0]2

It is an additive grading (additive quantum number), since the degree of a product is given by the addition of degrees

+		[0]0	[0]1	[0]2	[0]3
(0,0)	$\equiv [0]0$	[0]0	[0]1	[0]2	[0]3
(1,0)	$\equiv [0]1$	[0]1	[0]0	[0]3	[0]2
(0,1)	$\equiv [0]2$	[0]2	[0]3	[0]0	[0]1
(1,1)	$\equiv [0]3$	[0]3	[0]2	[0]1	[0]0

$$\approx \mathbb{Z}_2 \times \mathbb{Z}_2$$

$$T_{[0]j} = \frac{1}{2}\left(\frac{1}{2}\epsilon_{jkl} M^{kl} + \mathrm{i}\, M^{0j}\right); \quad j = 1,2,3$$
$$\bar{T}_{[0]j} = \frac{1}{2}\left(\frac{1}{2}\epsilon_{jkl} M^{kl} - \mathrm{i}\, M^{0j}\right); \quad j = 1,2,3.$$


Poincaré Algebra
$Z_2 X Z_2$
Extension
$I \supseteq Z_2 X Z_2$
$I = Z_2 X (Z_4n X Z_4n)$


03.30.2007

$$\mathbb{Z}_4 \times \mathbb{Z}_4$$

$$\{[0]0, [0]1, [0]2, [0]3\} \approx \mathbb{Z}_2 \times \mathbb{Z}_2$$

$+$	[0]0	[0]1	[0]2	[0]3	[1]0	[1]1	[1]2	[1]3	[2]0	[2]1	[2]2	[2]3	[3]0	[3]1	[3]2	[3]3
$(0,0) \equiv [0]0$	[0]0	[0]1	[0]2	[0]3	[1]0	[1]1	[1]2	[1]3	[2]0	[2]1	[2]2	[2]3	[3]0	[3]1	[3]2	[3]3
$(2,0) \equiv [0]1$	[0]1	[0]0	[0]3	[0]2	[1]1	[1]0	[1]3	[1]2	[2]1	[2]0	[2]3	[2]2	[3]1	[3]0	[3]3	[3]2
$(0,2) \equiv [0]2$	[0]2	[0]3	[0]0	[0]1	[1]2	[1]3	[1]0	[1]1	[2]2	[2]3	[2]0	[2]1	[3]2	[3]3	[3]0	[3]1
$(2,2) \equiv [0]3$	[0]3	[0]2	[0]1	[0]0	[1]3	[1]2	[1]1	[1]0	[2]3	[2]2	[2]1	[2]0	[3]3	[3]2	[3]1	[3]0
$(3,0) \equiv [1]0$	[1]0	[1]1	[1]2	[1]3	[0]1	[0]0	[0]3	[0]2	[3]3	[3]2	[3]1	[3]0	[2]2	[2]3	[2]0	[2]1
$(1,0) \equiv [1]1$	[1]1	[1]0	[1]3	[1]2	[0]0	[0]1	[0]2	[0]3	[3]2	[3]3	[3]0	[3]1	[2]3	[2]2	[2]1	[2]0
$(3,2) \equiv [1]2$	[1]2	[1]3	[1]0	[1]1	[0]3	[0]2	[0]1	[0]0	[3]1	[3]0	[3]3	[3]2	[2]0	[2]1	[2]2	[2]3
$(1,2) \equiv [1]3$	[1]3	[1]2	[1]1	[1]0	[0]2	[0]3	[0]0	[0]1	[3]0	[3]1	[3]2	[3]3	[2]1	[2]0	[2]3	[2]2
$(0,3) \equiv [2]0$	[2]0	[2]1	[2]2	[2]3	[3]3	[3]2	[3]1	[3]0	[0]2	[0]3	[0]0	[0]1	[1]1	[1]0	[1]3	[1]2
$(2,3) \equiv [2]1$	[2]1	[2]0	[2]3	[2]2	[3]2	[3]3	[3]0	[3]1	[0]3	[0]2	[0]1	[0]0	[1]0	[1]1	[1]2	[1]3
$(0,1) \equiv [2]2$	[2]2	[2]3	[2]0	[2]1	[3]1	[3]0	[3]3	[3]2	[0]0	[0]1	[0]2	[0]3	[1]3	[1]2	[1]1	[1]0
$(2,1) \equiv [2]3$	[2]3	[2]2	[2]1	[2]0	[3]0	[3]1	[3]2	[3]3	[0]1	[0]0	[0]3	[0]2	[1]2	[1]3	[1]0	[1]1
$(1,1) \equiv [3]0$	[3]0	[3]1	[3]2	[3]3	[2]2	[2]3	[2]0	[2]1	[1]1	[1]0	[1]3	[1]2	[0]3	[0]2	[0]1	[0]0
$(3,1) \equiv [3]1$	[3]1	[3]0	[3]3	[3]2	[2]3	[2]2	[2]1	[2]0	[1]0	[1]1	[1]2	[1]3	[0]2	[0]3	[0]0	[0]1
$(1,3) \equiv [3]2$	[3]2	[3]3	[3]0	[3]1	[2]0	[2]1	[2]2	[2]3	[1]3	[1]2	[1]1	[1]0	[0]1	[0]0	[0]3	[0]2
$(3,3) \equiv [3]3$	[3]3	[3]2	[3]1	[3]0	[2]1	[2]0	[2]3	[2]2	[1]2	[1]3	[1]0	[1]1	[0]0	[0]1	[0]2	[0]3

$$(\mathbb{Z}_4 \times \mathbb{Z}_4)/(\mathbb{Z}_2 \times \mathbb{Z}_2) \approx \mathbb{Z}_2 \times \mathbb{Z}_2$$

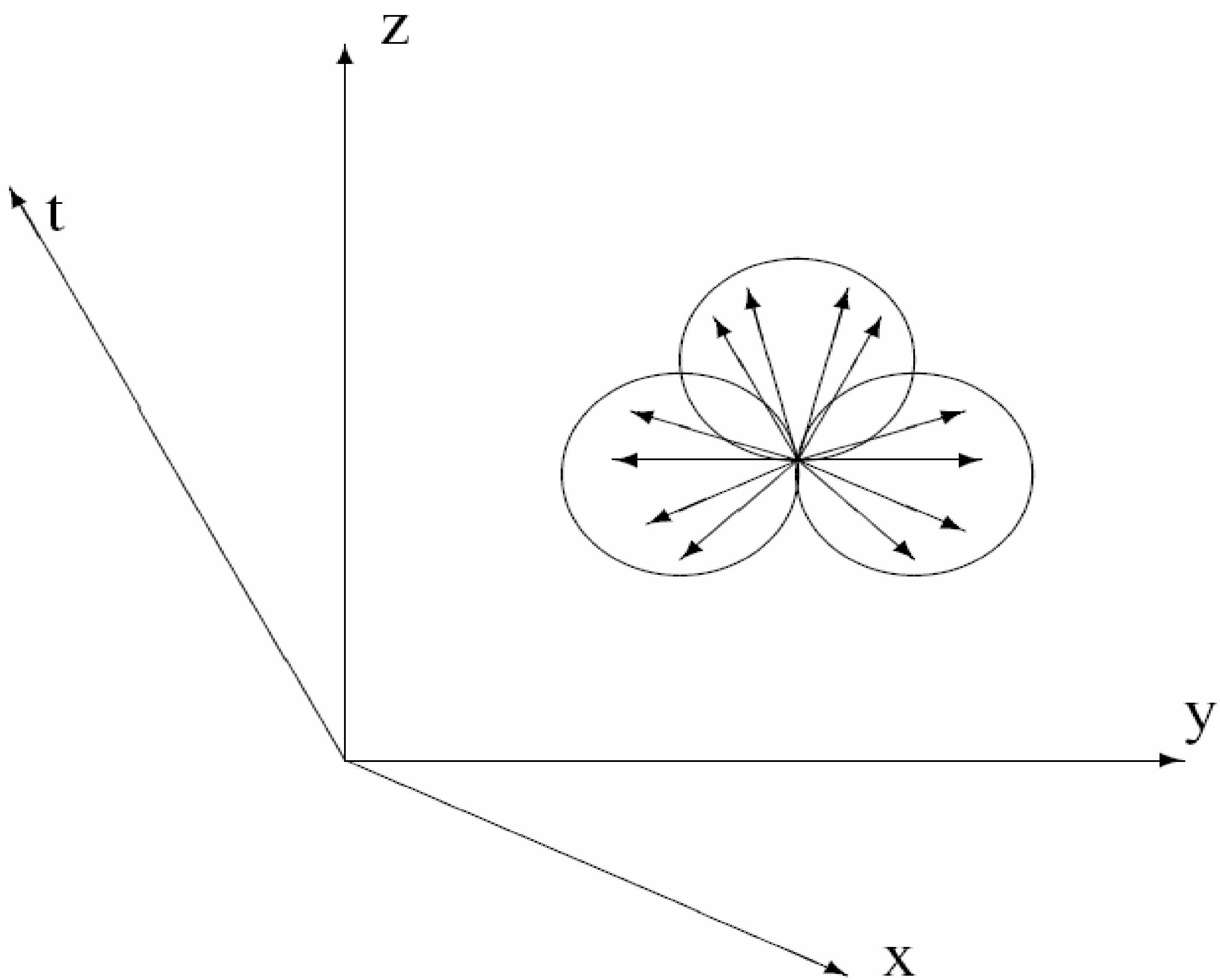
†	[0]	[1]	[2]	[3]
[0]	[0]	[1]	[2]	[3]
[1]	[1]	[0]	[3]	[2]
[2]	[2]	[3]	[0]	[1]
[3]	[3]	[2]	[1]	[0]

Clover Extensions:

a) Vector Scalar Clover Extension

Spin → ↓ Naive dim	$[T^2, R] = s(s+1)R$, $(1,0)$	$[\bar{T}^2, R] = t(t+1)R$, $(\frac{1}{2},\frac{1}{2})$, $(0,0)$	(s,t) $(0,1)$
1		$P_{[0]\mu}$	
2/3	$T_{[1]j}, T_{[2]j}, T_{[3]j}$	$E_{[1]0}, E_{[2]0}, E_{[3]0}, \bar{E}_{[1]0}, \bar{E}_{[2]0}, \bar{E}_{[3]0},$	$\bar{T}_{[1]j}, \bar{T}_{[2]j}, \bar{T}_{[3]j}$
1/3		$P_{[1]\mu}, P_{[2]\mu}, P_{[3]\mu}$	
0	$T_{[0]j}$		$\bar{T}_{[0]j}$

b) Minimal Vector Clover Extension

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c) Full Clover Extension

$U(1) \times U(1)$ subgroup

Spin → / ↓ Naive dim	$(1,0)$	$(\frac{1}{2},\frac{1}{2})$	$(0,0)$	$(0,1)$
1		$P_{[0]\mu}$		
2/3	$T_{[1]j}, T_{[2]j}, T_{[3]j}$		$E_{[1]0}, E_{[2]0}, E_{[3]0}, \bar{E}_{[1]0}, \bar{E}_{[2]0}, \bar{E}_{[3]0},$	$\bar{T}_{[1]j}, \bar{T}_{[2]j}, \bar{T}_{[3]j}$
1/3		$P_{[1]\mu}, P_{[2]\mu}, P_{[3]\mu}$		
0	$T_{[0]j}$		$E_{[0]0}, \bar{E}_{[0]0}$	$\bar{T}_{[0]j}$

$$\{Q_\alpha, \bar{Q}_{\dot\beta}\} = 2\,\sigma^{\mu}_{\alpha\dot\beta}\; P_{[0]\mu}. \qquad [P_{[1]\mu}, [P_{[2]\nu}, P_{[3]\rho}]] \sim P_{[0]\sigma}.$$

$$\begin{aligned}
[P_{[i]\mu}, P_{[j]\nu}] &= \mathrm{i}\; d_{i\dagger j}\;\; \sigma^P(r)_\mu{}^\rho\, \epsilon^P_{\rho\nu}\, T_{[i\dagger j]r}\, \beta_{i\dagger j} + \mathrm{i}\; d^*_{i\dagger j}\;\; \bar\sigma^P(\dot r)^\rho{}_\mu\, \epsilon^P_{\rho\nu}\, \bar T_{[i\dagger j]\dot r}\, \beta_{i\dagger j} + \\
&\quad -\frac{\mathrm{i}}{2}\epsilon_{ijk}\, f_k\, \epsilon^P_{\mu\nu}\, E_{[i\dagger j]0}\, \beta_{i\dagger j} - \frac{\mathrm{i}}{2}\epsilon_{ijk}\, f^*_k\, \epsilon^P_{\mu\nu}\, \bar E_{[i\dagger j]0}\, \beta_{i\dagger j};\;\; i \neq j, \\
[P_{[c]\mu}, P_{[c]\nu}] &= 0, \\
[T_{[k]r}\, \beta_k, P_{[l]\mu}] &= -\mathrm{i}\, \delta_{kl}\, e_k\, \sigma^P(r)_\mu{}^\nu P_{[0]\nu}, \\
[\bar T_{[k]\dot r}\, \beta_k, P_{[l]\mu}] &= -\mathrm{i}\; \delta_{kl}\, e^*_k\, \bar\sigma^P(\dot r)^\nu{}_\mu P_{[0]\nu}, \\
[E_{[k]0}\, \beta_k, P_{[l]\mu}] &= \frac{\mathrm{i}}{2}\delta_{kl}\, h_k\, P_{[0]\mu}, \\
[\bar E_{[k]0}\, \beta_k, P_{[l]\mu}] &= \frac{\mathrm{i}}{2}\delta_{kl}\, h^*_k\, P_{[0]\mu}.
\end{aligned}$$

$$\begin{aligned}
[T_{[0]i}, a_{[c]s}] &= -\mathrm{i}\; \sigma^a(i)_s{}^t\, a_{[c]t}, \\
[T_{[0]i}, a_{[c]}{}^s] &= \mathrm{i}\; a_{[f]}{}^t\, \sigma^a(i)_t{}^s, \\
[\bar T_{[0]i}, \bar a_{[c]\dot s}] &= -\mathrm{i}\; \bar a_{[c]\dot t}\, \bar\sigma^{\bar a}(i)^{\dot t}{}_{\dot s}, \\
[\bar T_{[0]i}, \bar a_{[c]}{}^{\dot s}] &= \mathrm{i}\; \bar\sigma^{\bar a}(i)^{\dot s}{}_{\dot t}\, \bar a_{[c]}{}^{\dot t}.
\end{aligned}$$

$$\begin{aligned}
[E_{[0]0}, \bar E_{[0]0}] &= 0, \\
[E_{[0]0}, P_{[i]\nu}] &= l_i\, P_{[i]\nu}, & [\bar E_{[0]0}, P_{[i]\nu}] &= -l^*_i\, P_{[i]\nu}, \\
[E_{[0]0}, T_{[k]a}] &= -l_k\, T_{[k]a}, & [\bar E_{[0]0}, T_{[k]a}] &= l^*_k\, T_{[k]a}, \\
[E_{[0]0}, \bar T_{[k]a}] &= -l_k\, \bar T_{[k]a}, & [\bar E_{[0]0}, \bar T_{[k]a}] &= l^*_k\, \bar T_{[k]a}, \\
[E_{[0]0}, E_{[k]0}] &= -l_k\, E_{[k]0}, & [\bar E_{[0]0}, E_{[k]0}] &= l^*_k\, E_{[k]0}, \\
[E_{[0]0}, \bar E_{[k]0}] &= -l_k\, \bar E_{[k]0}, & [\bar E_{[0]0}, \bar E_{[k]0}] &= l^*_k\, \bar E_{[k]0},
\end{aligned}$$

$$\bar\sigma^{\bar a}(i) = (\sigma^a(i))^\dagger,$$

$$l_1 + l_2 + l_3 = 0; \quad l_1, l_2, l_3 \in \mathbb{C}.$$

$$a_{[c]}{}^{u} = \epsilon^{a\ \ us}\, a_{[c]s}, \qquad \bar{a}_{[c]}{}^{\dot{u}} = \bar{\epsilon}^{\bar{a}\ \ \dot{u}\dot{s}}\, \bar{a}_{[c]\dot{s}}.$$

$\sigma^T(1)$	$\sigma^T(2)$	$\sigma^T(3)]$
$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ -1 & 0 & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

$\sigma^P(1)$	$\sigma^P(2)$	$\sigma^P(3)$
$\frac{1}{2}\begin{bmatrix} 0 & -\mathrm{i} & 0 & 0 \\ -\mathrm{i} & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$	$\frac{1}{2}\begin{bmatrix} 0 & 0 & -\mathrm{i} & 0 \\ 0 & 0 & 0 & 1 \\ -\mathrm{i} & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{bmatrix}$	$\frac{1}{2}\begin{bmatrix} 0 & 0 & 0 & -\mathrm{i} \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ -\mathrm{i} & 0 & 0 & 0 \end{bmatrix}$

The constants $d_i,\ f_i,\ e_i,\ h_i$ for $i = 1, 2, 3$ satisfy:

$$d_1 e_1 + d_2 e_2 + d_3 e_3 = d_1^* e_1^* + d_2^* e_2^* + d_3^* e_3^*,$$
$$f_1 h_1 + f_1^* h_1^* = (d_2 e_2 + d_2^* e_2^*) - (d_3 e_3 + d_3^* e_3^*),$$
$$f_2 h_2 + f_2^* h_2^* = (d_3 e_3 + d_3^* e_3^*) - (d_1 e_1 + d_1^* e_1^*),$$
$$f_3 h_3 + f_3^* h_3^* = (d_1 e_1 + d_1^* e_1^*) - (d_2 e_2 + d_2^* e_2^*).$$

The constants $d_i,\ f_i,\ e_i,\ h_i$ for $i = 1, 2, 3$ satisfy:

$$\begin{aligned} d_1 e_1 + d_2 e_2 + d_3 e_3 &= d_1^* e_1^* + d_2^* e_2^* + d_3^* e_3^*, \\ f_1 h_1 + f_1^* h_1^* &= (d_2 e_2 + d_2^* e_2^*) - (d_3 e_3 + d_3^* e_3^*), \\ f_2 h_2 + f_2^* h_2^* &= (d_3 e_3 + d_3^* e_3^*) - (d_1 e_1 + d_1^* e_1^*), \\ f_3 h_3 + f_3^* h_3^* &= (d_1 e_1 + d_1^* e_1^*) - (d_2 e_2 + d_2^* e_2^*). \end{aligned}$$

$$\begin{aligned} [P_{[1]\mu}, [P_{[2]\nu}, P_{[3]\rho}]] \;\; = \;\; & -[d_1\, e_1\, \sigma^P(r)_\nu{}^\sigma\, \epsilon^P_{\sigma\rho}\, \sigma^P(r)_\mu{}^\xi + \\ & + d_1^*\, e_1^*\, \bar{\sigma}^P(\dot{r})^\sigma{}_\nu\, \epsilon^P_{\sigma\rho}\, \bar{\sigma}^P(\dot{r})^\xi{}_\mu + \\ & + \frac{1}{4}(f_1 h_1 + f_1^* h_1^*)\, (\epsilon^P_{\nu\rho}\, \delta^\sigma_\mu)]\, P_{[0]\sigma}. \end{aligned}$$

d) VSC Extension with su(2) enhancement?

Spin → ↓ Naive dim	$(1,0)$	$(\frac{1}{2},\frac{1}{2})$ $(0,0)$	$(0,1)$
1		$P_{[0]\mu}$	
2/3	$T_{[1]j}, T_{[2]j}, T_{[3]j}$	$E_{[1]0}, E_{[2]0}, E_{[3]0}, \bar{E}_{[1]0}, \bar{E}_{[2]0}, \bar{E}_{[3]0},$	$\bar{T}_{[1]j}, \bar{T}_{[2]j}, \bar{T}_{[3]j}$
1/3		$P_{[1]\mu}, P_{[2]\mu}, P_{[3]\mu}$	
0	$T_{[0]j}$		$\bar{T}_{[0]j}$

$$[G_{a[\mu]}, G_{b[\nu]}] = \frac{i}{2}\zeta_a(\mu)_b{}^c G_{c[\mu\dagger\nu]}, \quad [G_{a[\mu]}, P_{[i]\nu}] = \frac{i}{2}\lambda_a(\mu)_i{}^{i\dagger\mu} P_{[i\dagger\mu]\nu},$$

$$[G_{a[\mu]}, T_{[i]s}] = \frac{i}{2}\gamma_a(\mu)_i{}^{i\dagger\mu} T_{[i\dagger\mu]s}, \quad [G_{a[\mu]}, \bar{T}_{[i]s}] = \frac{i}{2}\gamma^*_a(\mu)_i{}^{i\dagger\mu}\, \bar{T}_{[i\dagger\mu]s},$$

$$[G_{a[\mu]}, E_{[i]0}] = \frac{i}{2}\rho_a(\mu)_i{}^{i\dagger\mu}\, E_{[i\dagger\mu]0}, \quad [G_{a[\mu]}, \bar{E}_{[i]0}] = \frac{i}{2}\rho^*_a(\mu)_i{}^{i\dagger\mu}\, \bar{E}_{[i\dagger\mu]0}.$$

Observe that due to the absence of generators of class [0] in dimensions 1/3 and 2/3, we are forced to require:

$$[G_{a[i]}, T_{[i]s}] = 0, \quad [G_{a[i]}, \bar{T}_{[i]s}] = 0,$$
$$[G_{a[i]}, E_{[i]0}] = 0, \quad [G_{a[i]}, \bar{E}_{[i]0}] = 0.$$

$$d_i\, e_i = 0, \quad f_i\, h_i = f_j\, h_j, \quad \text{for all} \quad i, j = 1, 2, 3.$$

$\lambda_a(1)$	$\lambda_a(2)$	$\lambda_a(3)$
$C_{a_1}\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \end{bmatrix}$	$C_{a_2}\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$	$C_{a_3}\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

$$f_i = h_i = 1, \zeta(i)_j{}^k = \lambda(i)_j{}^k = \rho(i)_j{}^k = \rho^*(i)_j{}^k = \epsilon_{ijk}\,\beta_k.$$

Spin → ↓ Naive dim	$(1,0)$	$(\frac{1}{2},\frac{1}{2})$	$(0,0)$	$(0,1)$
1		$P_{[0]\mu}$		
2/3			$E_{[1]0}, E_{[2]0}, E_{[3]0}, \bar{E}_{[1]0}, \bar{E}_{[2]0}, \bar{E}_{[3]0}$	
1/3		$P_{[1]\mu}, P_{[2]\mu}, P_{[3]\mu}$		
0	$T_{[0]j}$		$G_{[1]0}, G_{[2]0}, G_{[3]0}$	$\bar{T}_{[0]j}$

The $su(2)$ scalar Clover Extension, $j = 1, 2, 3$; $\mu = 0, 1, 2, 3$.


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e) The $su(3)$-scalar clover extension

$$G_{[c,u]} = \begin{bmatrix} G_{[0,1]} & G_{[0,2]} \\ G_{[1,1]} & G_{[1,2]} \\ G_{[2,1]} & G_{[2,2]} \\ G_{[3,1]} & G_{[3,2]} \end{bmatrix} = \begin{bmatrix} G_3 & G_8 \\ G_6 & G_7 \\ G_4 & G_5 \\ G_1 & G_2 \end{bmatrix}$$

$$P_{[c,u]\mu} = \begin{bmatrix} P_{[0,1]\mu} & P_{[0,2]\mu} \\ P_{[1,1]\mu} & P_{[1,2]\mu} \\ P_{[2,1]\mu} & P_{[2,2]\mu} \\ P_{[3,1]\mu} & P_{[3,2]\mu} \end{bmatrix}$$

$$E_{[c,u]} = \begin{bmatrix} E_{[0,1]} & E_{[0,2]} \\ E_{[1,1]} & E_{[1,2]} \\ E_{[2,1]} & E_{[2,2]} \\ E_{[3,1]} & E_{[3,2]} \end{bmatrix}$$

$$[G_{[c,u]}, G_{[e,v]}] = \mathrm{i}\ \mathrm{f}_{[c,u],[e,v]}^{\ [h,w]}\, G_{[h,w]}.$$

$$[G_{[c,u]}, P_{[e,v]\nu}] = \mathrm{i}\ \mathrm{f}_{[c,u],[e,v]}^{\ [h,w]}\, P_{[h,w]\nu},$$

$$[G_{[c,u]}, E_{[e,v]}] = \mathrm{i}\ \mathrm{f}_{[c,u],[e,v]}^{\ [h,w]}\, E_{[h,w]}.$$

$$
\begin{aligned}
[P_{[c,u]\mu}, P_{[e,v]\nu}] &= \mathrm{i}\; H\, \mathrm{f}_{[c,u],[e,v]}^{\;[h,w]}\, \epsilon^{P}_{\mu\nu}\, E_{[h,w]} + \mathrm{i}\; H^{*}\, \mathrm{f}_{[c,u],[e,v]}^{\;[h,w]}\, \epsilon^{P}_{\mu\nu}\, \bar{E}_{[h,w]}, \\
[P_{[c,u]\mu}, E_{[e,v]}] &= \mathrm{i}\; J\, \delta_{c,e}\, \delta_{u,v}\, P_{[0]\mu}, \\
[P_{[c,u]\mu}, \bar{E}_{[e,v]}] &= \mathrm{i}\; J^{*}\, \delta_{c,e}\, \delta_{u,v}\, P_{[0]\mu}
\end{aligned}
$$

Spin → ↓ Naive dim	$(1,0)$	$(\frac{1}{2},\frac{1}{2})$	$(0,0)$	$(0,1)$
1		$P_{[0]\mu}$		
2/3			$E_{[c,u]}, \bar{E}_{[c,u]}$	
1/3		$P_{[c,u]\mu}$		
0	$T_{[0]j}$		$G_{[c,u]}$	$\bar{T}_{[0]j}$

The $su(3)$ scalar clover ext., $j = 1,2,3;\; c,\mu = 0,1,2,3;\; u = 1,2$

f) The su(3) Full Clover Ext.

$$G_{[c,u]} = \begin{bmatrix} G_{[0,1]} & G_{[0,2]} \\ G_{[1,1]} & G_{[1,2]} \\ G_{[2,1]} & G_{[2,2]} \\ G_{[3,1]} & G_{[3,2]} \end{bmatrix} = \begin{bmatrix} G_3 & G_8 \\ G_6 & G_7 \\ G_4 & G_5 \\ G_1 & G_2 \end{bmatrix}$$

$$P_{t[c,u]\mu} = \begin{bmatrix} P_{t[0,1]\mu} & P_{t[0,2]\mu} \\ P_{t[1,1]\mu} & P_{t[1,2]\mu} \\ P_{t[2,1]\mu} & P_{t[2,2]\mu} \\ P_{t[3,1]\mu} & P_{t[3,2]\mu} \end{bmatrix}$$

$$T_{t[c,u]r} = \begin{bmatrix} T_{t[0,1]r} & T_{t[0,2]r} \\ T_{t[1,1]r} & T_{t[1,2]r} \\ T_{t[2,1]r} & T_{t[2,2]r} \\ T_{t[3,1]r} & T_{t[3,2]r} \end{bmatrix}$$

$$E_{t[c,u]} = \begin{bmatrix} E_{t[0,1]} & E_{t[0,2]} \\ E_{t[1,1]} & E_{t[1,2]} \\ E_{t[2,1]} & E_{t[2,2]} \\ E_{t[3,1]} & E_{t[3,2]} \end{bmatrix}$$

Spin → ↓ Naive dim	$(1,0)$	$(\frac{1}{2},0)$	$(\frac{1}{2},\frac{1}{2})$ $-\ -\ -\ -$ $(0,0)$	$(0,\frac{1}{2})$	$(0,1)$
1			$P_{[0]\mu}$ $-\ -\ -\ -$ $h_{t[i]}, \bar{h}_{t[i]}$		
$\frac{4}{6}$	$T_{t[c,u]r}$		$-$ $-\ -\ -\ -$ $E_{t[c,u]}, \bar{E}_{t[c,u]}$		$\bar{T}_{t[c,u]\dot{r}}$
$\frac{2}{6}$			$P_{t[c,u]\mu}$ $-\ -\ -\ -$ $B_{t[i]}, \bar{B}_{t[i]}$		
0	$T_{[0]j}$		$-\ -\ -\ -\ -\ -$ $G_{[c,u]}, G'_{t[0]}$		$\bar{T}_{[0]j}$

$su(3)$-FCE, $i,j = 1,2,3$; $c,\mu = 0,1,2,3$.

g) FC + Susy Extension, $i, j = 1, 2, 3$; $c, \mu = 0, 1, 2, 3$

Spin → ↓ Naive dim	$(1,0)$	$(\frac{1}{2},0)$	$(\frac{1}{2},\frac{1}{2})$ — — — — $(0,0)$	$(0,\frac{1}{2})$	$(0,1)$
1			$P_{[0]\mu}$ — — — — —		
2/3	$T_{[i]j}$		— — — — — $E_{[i]0}, \bar{E}_{[i]0}$		$\bar{T}_{[i]j}$
3/6		$Q_{\alpha[c]s}$		$\bar{Q}_{\alpha[c]s}$	
1/3			$P_{[i]\mu}$ — — — — —		
0	$T_{[0]j}$		— — — — — $E_{[0]0}, \bar{E}_{[0]0}$		$\bar{T}_{[0]j}$

$$
\begin{aligned}
[T_{[0]i}, Q_{\alpha[c]s}] &= -\mathrm{i}\; \sigma^{Q}(i)_{\alpha}{}^{\beta}\, Q_{\beta[c]s} \\
[\bar{T}_{[0]i}, Q_{\alpha[c]s}] &= 0 \\
[T_{[0]i}, \bar{Q}_{\dot\alpha[c]s}] &= 0 \\
[\bar{T}_{[0]i}, \bar{Q}_{\dot\alpha[c]s}] &= -\mathrm{i}\; \bar{Q}_{\dot\beta[c]s}\, \bar{\sigma}^{\bar{Q}}(i)^{\dot\beta}{}_{\dot\alpha}.
\end{aligned}
$$

$$
\begin{aligned}
\epsilon^{Q\;\alpha\beta}\, Q_{\beta[c]s} &= Q^{\alpha}{}_{[c]s} \\
\epsilon^{Q}_{\alpha\beta}\, Q^{\beta}{}_{[c]s} &= Q_{\alpha[c]s} \\
\bar{\epsilon}^{\bar{Q}\;\dot\alpha\dot\beta}\, \bar{Q}_{\dot\beta[c]s} &= \bar{Q}^{\dot\alpha}{}_{[c]s} \\
\bar{\epsilon}^{\bar{Q}}_{\dot\alpha\dot\beta}\, \bar{Q}^{\dot\beta}{}_{[c]s} &= \bar{Q}_{\dot\alpha[c]s}.
\end{aligned}
$$

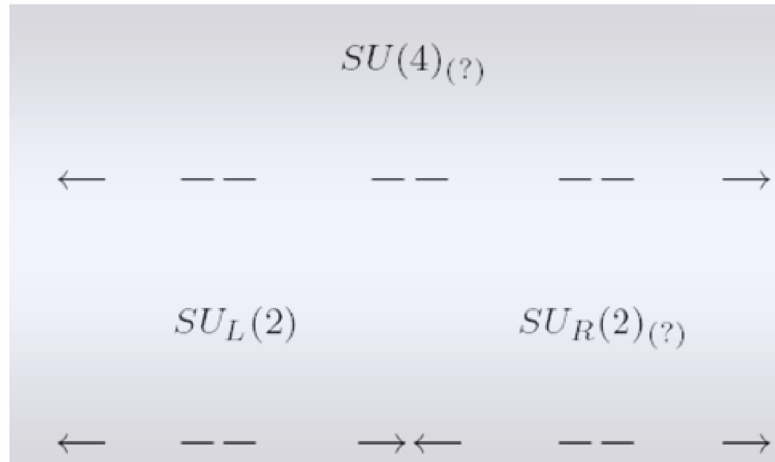
$$
\epsilon^{Q\;\alpha\beta} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}^{\alpha\beta}, \qquad \epsilon^{Q}_{\alpha\beta} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}_{\alpha\beta},
$$

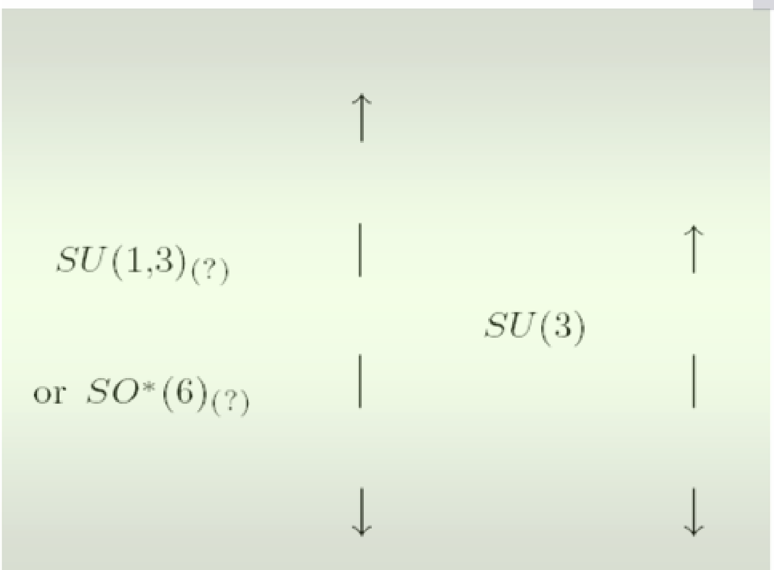
$$
\bar{\epsilon}^{\bar{Q}\;\dot\alpha\dot\beta} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}^{\dot\alpha\dot\beta}, \qquad \bar{\epsilon}^{\bar{Q}}_{\dot\alpha\dot\beta} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}_{\dot\alpha\dot\beta},
$$

$\sigma^{Q}(1)$	$\sigma^{Q}(2)$	$\sigma^{Q}(3)$	$\bar{\sigma}^{\bar{Q}}(1)$	$\bar{\sigma}^{\bar{Q}}(2)$	$\bar{\sigma}^{\bar{Q}}(3)$
$\frac{i}{2}\beta_1\sigma_1$	$-\frac{i}{2}\beta_2\sigma_2$	$-\frac{i}{2}\beta_3\sigma_3$	$\frac{i}{2}\beta_1\sigma_1$	$\frac{i}{2}\beta_2\sigma_2$	$\frac{i}{2}\beta_3\sigma_3$

$$[\![Q_{\alpha(c)s}, \bar{Q}_{\dot\beta(d)t}]\!] = 2\,\delta_{cd}\,\delta_{st}\,\beta_{(c)s+(d)t}\,\sigma^{\mu}_{Q}(c)_{\alpha\dot\beta}\,P_{(0)\mu}$$

$$[\![G_{a(\rho)s}, Q_{\alpha(c)t}]\!] = \frac{1}{2}\,\beta_{(\rho)s+(c)t-(c\dagger\rho)t}\,\lambda_a(\rho, s, t)_c{}^{c\dagger\rho} Q_{\alpha(c\dagger\rho)t}.$$





$Q_{\alpha(0)0}$	$Q_{\alpha(0)1}$	$Q_{\alpha(0)2}$	$Q_{\alpha(0)3}$
$Q_{\alpha(1)0}$	$Q_{\alpha(1)1}$	$Q_{\alpha(1)2}$	$Q_{\alpha(1)3}$
$Q_{\alpha(2)0}$	$Q_{\alpha(2)1}$	$Q_{\alpha(2)2}$	$Q_{\alpha(2)3}$
$Q_{\alpha(3)0}$	$Q_{\alpha(3)1}$	$Q_{\alpha(3)2}$	$Q_{\alpha(3)3}$

$$\breve{\lambda}_a = -\left[\begin{array}{c|ccc} 0 & 0 & 0 & 0 \\ \hline 0 & & & \\ 0 & & \lambda_a & \\ 0 & & & \end{array}\right]$$

$$\lambda_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad \lambda_8 = \frac{1}{\sqrt{3}} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{bmatrix},$$

$$\lambda_6 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}, \quad \lambda_7 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -\mathrm{i} \\ 0 & \mathrm{i} & 0 \end{bmatrix},$$

$$\lambda_4 = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}, \quad \lambda_5 = \begin{bmatrix} 0 & 0 & -\mathrm{i} \\ 0 & 0 & 0 \\ \mathrm{i} & 0 & 0 \end{bmatrix},$$

$$\lambda_1 = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad \lambda_2 = \begin{bmatrix} 0 & -\mathrm{i} & 0 \\ \mathrm{i} & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}.$$

g) su(3) Trefoil Extension

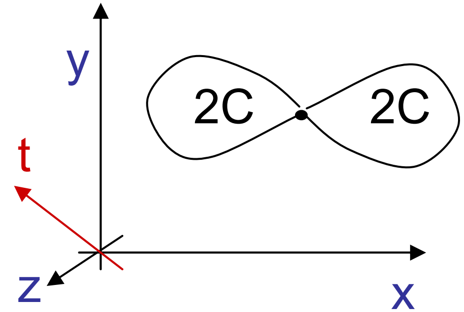
Spin → ↓ Naive dim	$(1,0)$	$(\frac{1}{2},0)$	$(\frac{1}{2},\frac{1}{2})$ – – – – $(0,0)$	$(0,\frac{1}{2})$	$(0,1)$
1			$P_{[0]\mu}$ – – – – $h_{t[i]}, \bar{h}_{t[i]}$		
$\frac{5}{6}$		$S_{\alpha[i]t}$		$\bar{S}_{\alpha[i]t}$	
$\frac{4}{6}$	$T_{t[c,u]r}$		– – – – – $E_{t[c,u]}, \bar{E}_{t[c,u]}$		$\bar{T}_{t[c,u]\dot{r}}$
$\frac{3}{6}$		$Q_{\alpha[i]t}$		$\bar{Q}_{\alpha[i]t}$	
$\frac{2}{6}$			$P_{t[c,u]\mu}$ – – – – $B_{t[i]}, \bar{B}_{t[i]}$		
$\frac{1}{6}$		$R_{\alpha[i]t}$		$\bar{R}_{\alpha[i]t}$	
0	$T_{[0]j}$		– – – – – – $G_{[c,u]}, G'_{t[0]}$		$\bar{T}_{[0]j}$

ble 15: $su(3)$-Trefoil Extension, $i,j = 1,2,3$; $c,\mu = 0,1,2,3$.

Summary:

Supersymmetry provides a Z_2-graded extension, in which two spinor charges combine to produce a space-time translation:

$$[Q_\alpha, \bar{Q}_{\dot{\beta}}] = 2\,\sigma^{\mu}_{\alpha\dot{\beta}}\; P_{[0]\mu},$$

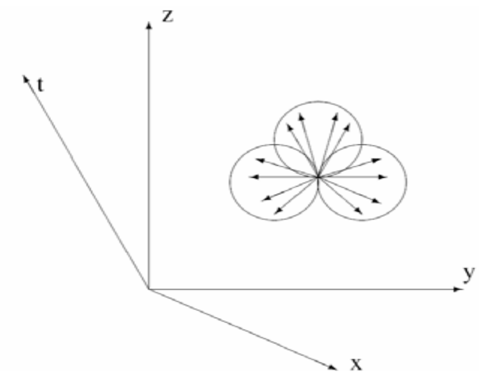


The Clover extensions are Z_4XZ_4-graded extensions, in which three vector charges combine to produce a translation:

$$[P_{[1]\mu}, [P_{[2]\nu}, P_{[3]\rho}]] = -[d_1\, e_1\, \sigma^P(r)_\nu{}^\sigma\, \epsilon^P_{\sigma\rho}\, \sigma^P(r)_\mu^\xi + \quad +d_1^*\, e_1^*\, \bar{\sigma}^P(\dot{r})^\sigma{}_\nu\, \epsilon^P_{\sigma\rho}\, \bar{\sigma}^P(\dot{r})^\xi{}_\mu + \\ +\frac{1}{4}(f_1 h_1 + f_1^* h_1^*)\,(\epsilon^P_{\nu\rho}\, \delta^\sigma_\mu)]\; P_{[0]\sigma}.$$

$Z_2X(Z_4XZ_4)$- graded extensions:
Susy, Internal Symmetries,
Dark Energy, Dim. confinement

u(1)xu(1), su(2), su(3)?




26.12.2005