2D Solitons in Dissipative Media

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Nonlinear Waves
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Outline

Complex Cubic-Quintic Ginzburg-Landau Equation (CCQGLE)

- CCQGLE

$$\partial_t A = \epsilon A + (b_1 + ic_1)\nabla_\perp^2 A - (b_3 - ic_3)|A|^2 A - (b_5 - ic_5)|A|^4 A$$

- Canonical equation governing the weakly nonlinear behavior of dissipative systems
- $\nabla_\perp^2$ −transverse Laplacian for radially symmetric beams, $A(x, y; t)$−envelope field, t−cavity number
- ϵ−linear loss/gain, b_1−angular spectral filtering, $c_1 = 0.5$− diffraction coefficient, b_3−nonlinear gain/loss, $c_3 = 1$−nonlinear dispersion, b_5−saturation of the nonlinear gain/loss, c_5−saturation of the nonlinear refractive index
- Akhmediev et. al. [1] new classes: pulsating, creeping, snaking, chaotical

Previous Numerical Simulations on 1D CCQGLE

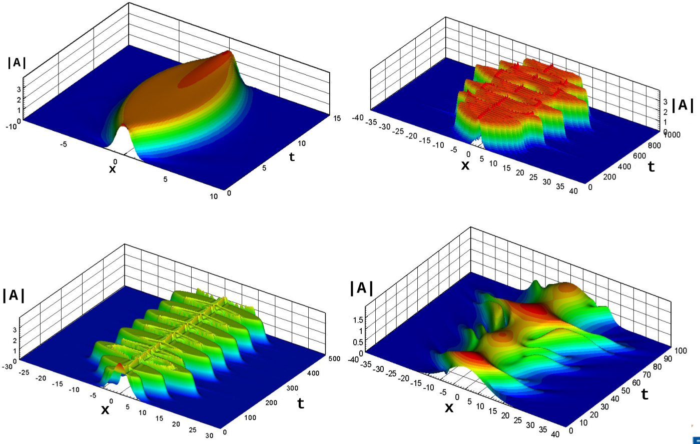

Figure: Pulsating, Snaking, Creeping, Chaotical

Hamiltonian Systems $\rightarrow$ No Hopf Bifurcations

- Five classes of solutions that are not stationary in time
- Don't exist as stable structures in Hamiltonian systems
- Envelopes exhibit complicated temporal dynamics and are unique to dissipative systems
- Dissipation allows the occurrence of Hopf and it leads to the various classes of pulsating solitons in CCQGLE

2D Fourier Spectral Method

- Fourier
  $\mathcal{F}(u)(k_x, k_y) = \widehat{u}(k_x, k_y) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{-i(k_x x + k_y y)} u(x, y)\, dxdy$
- inverse Fourier
  $\mathcal{F}^{-1}(\widehat{u})(x, y) = u(x, y) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{i(k_x x + k_y y)} \widehat{u}(k_x, k_y)\, dk_x dk_y$
- PDE $\Rightarrow$ ODE
  $\widehat{A}_t = \alpha(k_x, k_y)\widehat{A} + \beta\widehat{|A|^2 A} + \gamma\widehat{|A|^4 A}$
- $\alpha(k_x, k_y) = \epsilon - (b_1 + ic_1)(k_x^2 + k_y^2)$, $\beta = -(b_3 - ic_3)$,
  $\gamma = -(b_5 - ic_5)$

Spatial Discretization (Discrete Fourier Transform)

- Rectangular Mesh $\Omega = [-L/2, L/2] \times [-L/2, L/2]$ into $n \times n$ uniformly spaced grid points $X_{ij} = (x_i, y_j)$ with $\Delta x = \Delta y = L/n$, and $A(X_{ij}) = A_{ij}$
- 2DFT
  $\widehat{A}_{k_x k_y} = \Delta x \Delta y \sum_{i=1}^{n} \sum_{j=1}^{n} e^{-i(k_x x_i + k_y y_j)} A_{ij}, \;\; k_x, k_y = -\frac{n}{2} + 1, \cdots, \frac{n}{2}$
- inverse 2DFT
  $A_{ij} = \frac{1}{(2\pi)^2} \sum_{k_x=-n/2+1}^{n/2} \sum_{k_y=-n/2+1}^{n/2} e^{i(k_x x_i + k_y y_j)} \widehat{A}_{k_x k_y}, \; i, j = 1, 2, \cdots, n$

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Temporal Discretization

- Explicit scheme for the nonlinear part, and exact solution for the linear part $\widehat{A}(t) = \widehat{A(x,y;0)}e^{\alpha(k_x,k_y)t}$
- Initializing $\widehat{A}^n = \widehat{A}(t_n) \Rightarrow$
$$\mathcal{N}_3 = \mathcal{F}\left(\left|\mathcal{F}^{-1}(\widehat{A}^n)\right|^2 \mathcal{F}^{-1}(\widehat{A}^n)\right), \mathcal{N}_5 = \mathcal{F}\left(\left|\mathcal{F}^{-1}(\widehat{A}^n)\right|^4 \mathcal{F}^{-1}(\widehat{A}^n)\right)$$
- 4 step AB, or 4th order RK
$$\widehat{A}^{n+1} = \widehat{A}^n e^{\alpha(k_x,k_y)t} + \frac{\Delta t}{24}\left[55f(\widehat{A}^n) - 59f(\widehat{A}^{n-1}) + 37f(\widehat{A}^{n-2}) - 9f(\widehat{A}^{n-3})\right]$$
- $f(\widehat{A}) = \beta\mathcal{N}_1 + \gamma\mathcal{N}_2$

IC

- Gaussian $A(x, y; 0) = A_0 e^{-r^2}$
- ring shape with rotating phase $A(x, y; 0) = A_0 r^m e^{-r^2} e^{im\theta}$
- $m-$ degree of vorticity, A_0- real amplitude, $\theta = \tan^{-1}\left(\frac{\sigma_y y}{\sigma_x x}\right)$
- widths either circular or elliptic are controlled by $r = \sqrt{(\sigma_x x)^2 + (\sigma_y y)^2}$

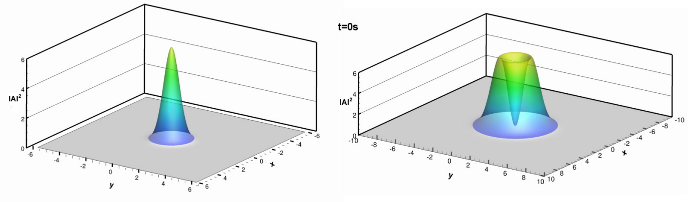


Figure: Initial shapes of solitons. Left: Gaussian, Right: Ring with vorticity $m = 1$.

System's Parameters

- Initial parameters
- Monitor energy
  $Q(t) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} |A(x, y; t)|^2 \, dxdy = \sum_{i=1}^{n} \sum_{j=1}^{n} |A_{ij}|^2 \Delta x \Delta y$

2D solitons	ϵ	b_1	c_1	b_3	c_3	b_5	c_5
Stationary	-0.045	0.04	0.5	-0.21	1	0.03	-0.08
Vortex (spinning)	-0.1	0.1	0.5	-0.88	1	0.04	-0.02
Pulsating	-0.045	0.04	0.5	-0.37	1	0.05	-0.08
Exploding/Erupting	-0.1	0.125	0.5	-1	1	0.1	-0.6
Creeping	-0.1	0.101	0.5	-1.3	1	0.3	-0.101

Table: Initial sets of parameters for 2D solitons from which we start simulations [1]

Stationary Solitons

- circular Gaussian IC and stays radially symmetric, stable and uninteresting. $A_0 = 2.5$, and $\sigma_x = \sigma_y = 1$

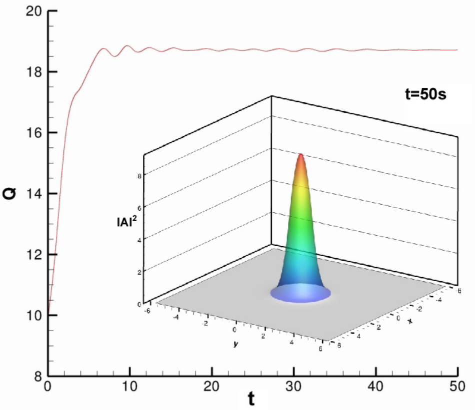


Figure: Energy is concentrated in the center of the domain

Ring Vortex (stable) Solitons

- Circular ring with rotating phase IC but different parameters, stable, it is spinning around its center. $A_0 = 2.5$, and $\sigma_x = \sigma_y = 1$

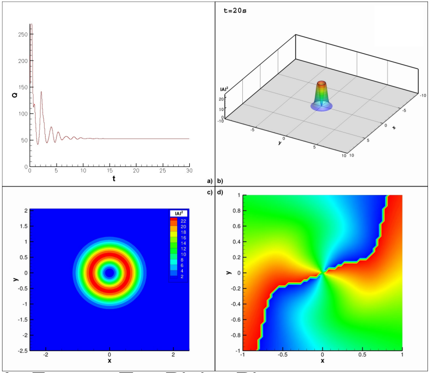


Figure: Top Left: Energy. Top Right: Ring vortex at $t = 20s$. Bottom Left: Contour plot of $|A|^2$. Bottom Right: Phase plot of θ at $t = 20s$

Ring Vortex (unstable) Solitons

- Circular Vortex it is spinning so much that breaks its symmetry
- changes into several bell-shaped solitons via multiple bifurcations, $A_0 = 3$, $\sigma_x = 0.15$, $\sigma_v = 0.15$

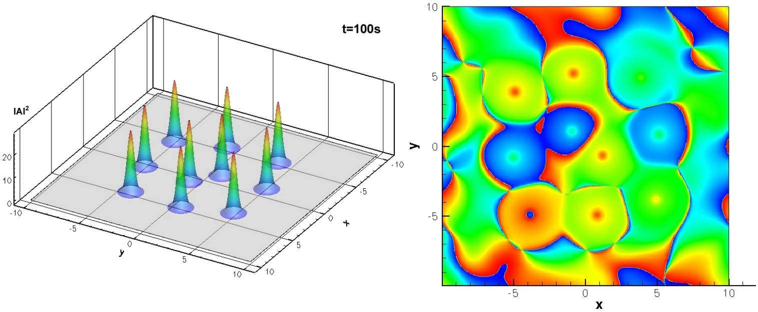


Figure: Left: 10 bell-shaped solitons due to defocusing. Right: phases are not spinning

Ring Vortex (stable) Solitons

- Elliptic stable, it is spinning around its center, and breaks symmetry but remains stable, $A_0 = 2.5$, $\sigma_x = 0.15$, $\sigma_y = 0.85$

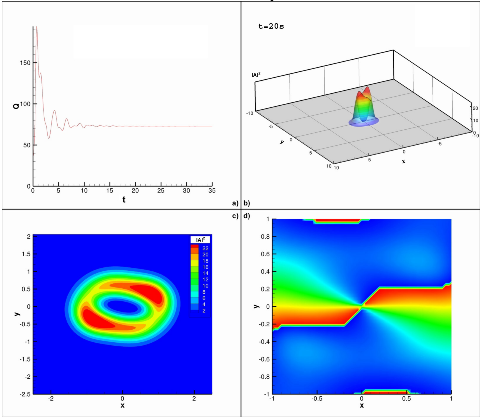
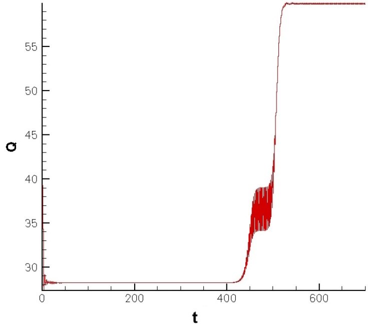


Figure: Two peaks appear on top

Pulsating Solitons (change stability)

- Gaussian IC, $A_0 = 5$, slightly elliptical, $\sigma_x = 0.8333$ and $\sigma_y = 0.9091$
- Pulsating similar to stationary initially but requires longer time to capture pulsations



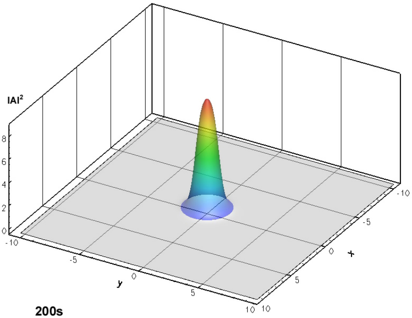
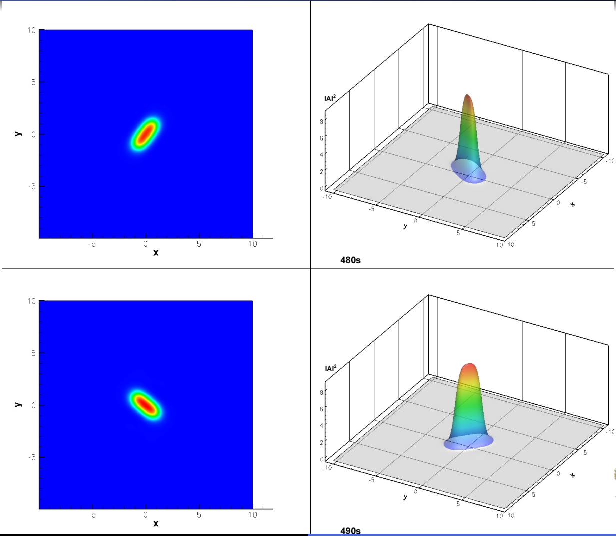
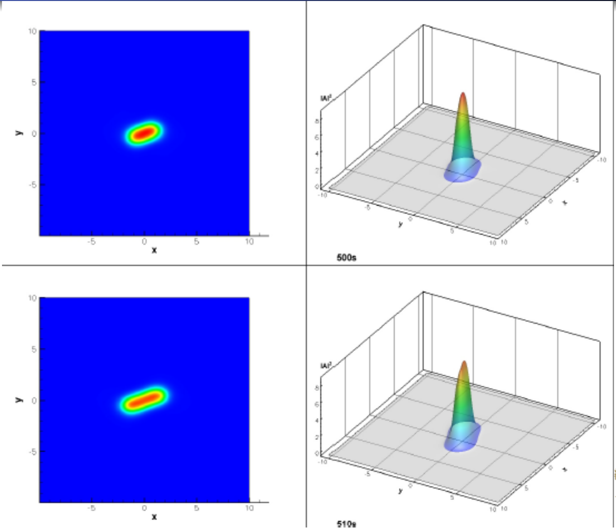


Figure: Left: Energy shows transitions. Right: No pulsations at $t = 200s$

Pulsating Phase at $t = 480s$, $t = 490s$



Pulsating Phase at $t = 500s$, $t = 510s$



Parameters for Exploding/Erupting

- Gaussian IC, $A_0 = 3.0$, and circular $\sigma_x = \sigma_y = 0.3$
- computed over 64 simulations within a 5 dimensional space by varying parameters one by one and looked for right $Q(t)$

	Parameters for ZEUS Simulations(Exploding c1=0.5 and c3=1)						
Parameters	**2*c1**	**c5**	**epsilon**	**-b3**	**b1**	**-b5**	**Job**
energy1	1.00000	-0.60000	-0.10000	1.00000	0.12500	-0.10000	test run
energy2	1.00000	-0.50000	-0.08000	1.40000	0.10000	-0.12500	2224
energy3	1.00000	-0.50000	-0.10000	1.00000	0.12500	-0.10000	2230
energy4	1.00000	-0.60000	-0.15000	1.00000	0.12500	-0.10000	2227
energy5	1.00000	-0.40000	-0.10000	1.00000	0.12500	-0.10000	2237
energy6	1.00000	-0.60000	-0.20000	1.00000	0.12500	-0.10000	2238
energy7	1.00000	-0.60000	-0.20000	1.20000	0.12500	-0.10000	2261
energy8	1.00000	-0.60000	-0.20000	1.00000	0.13000	-0.10000	2262
energy9	1.00000	-0.60000	-0.20000	1.00000	0.13500	-0.10000	2349
energy10	1.00000	-0.60000	-0.20000	0.80000	0.13000	-0.10000	2350
energy11	1.00000	-0.50000	-0.20000	0.80000	0.13500	-0.10000	2358
energy12	1.00000	-0.50000	-0.20000	0.80000	0.13500	-0.08000	2363
energy13	1.00000	-0.50000	-0.20000	0.80000	0.13000	-0.10000	2360
energy14	1.00000	-0.50000	-0.20000	0.60000	0.13500	-0.10000	2361
energy15	1.00000	-0.50000	-0.15000	0.80000	0.13500	-0.10000	2362
energy16	1.00000	-0.50000	-0.20000	0.90000	0.13500	-0.10000	2371
energy17	1.00000	-0.50000	-0.20000	0.90000	0.13500	-0.09000	2372
energy18	1.00000	-0.50000	-0.20000	0.90000	0.13000	-0.10000	2373
energy19	1.00000	-0.50000	-0.20000	0.85000	0.13500	-0.10000	2374
energy20	1.00000	-0.50000	-0.15000	0.90000	0.13500	-0.10000	2375
energy21	1.00000	-0.50000	-0.20000	0.90000	0.13500	-0.10000	2399
energy22	1.00000	-0.50000	-0.20000	0.90000	0.13500	-0.11000	2408
energy23	1.00000	-0.50000	-0.20000	0.90000	0.14000	-0.10000	2409
energy24	1.00000	-0.50000	-0.15000	0.90000	0.13500	-0.10000	2412
energy25	1.00000	-0.50000	-0.15000	0.90000	0.14000	-0.10000	2413
energy26	1.00000	-0.50000	-0.20000	0.90000	0.13500	-0.10000	2463
energy27	1.00000	-0.50000	-0.20000	0.90000	0.13500	-0.09500	2464
energy28	1.00000	-0.50000	-0.20000	0.90000	0.13000	-0.10000	2465
energy29	1.00000	-0.50000	-0.20000	0.87500	0.13500	-0.10000	2466
energy30	1.00000	-0.50000	-0.25000	0.90000	0.13500	-0.10000	2467
energy31	1.00000	-0.40000	-0.20000	0.90000	0.13500	-0.09500	2490
energy32	1.00000	-0.50000	-0.20000	0.90000	0.13500	-0.09000	2491
energy33	1.00000	-0.50000	-0.20000	0.90000	0.14000	-0.09500	2492
energy34	1.00000	-0.50000	-0.20000	0.85000	0.13500	-0.09500	2493
energy35	1.00000	-0.50000	-0.30000	0.90000	0.13500	-0.09500	2494
energy36	1.00000	-0.50000	-0.30000	0.90000	0.13500	-0.10500	2505
energy37	1.00000	-0.50000	-0.30000	0.90000	0.13500	-0.09000	2506
energy38	1.00000	-0.50000	-0.30000	0.90000	0.13750	-0.09500	2508

Energy for Exploding/Erupting

- Gaussian IC, $A_0 = 3.0$, and circular $\sigma_x = \sigma_y = 0.3$
- Exploding: look for high bursts of energy

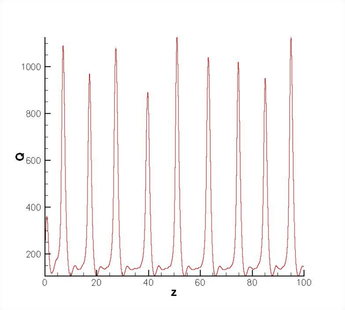


Figure: Energy is periodic with high bursts almost every 12s

Exploding/Erupting

- Initial soliton is smooth, then circular waves appear and grow.

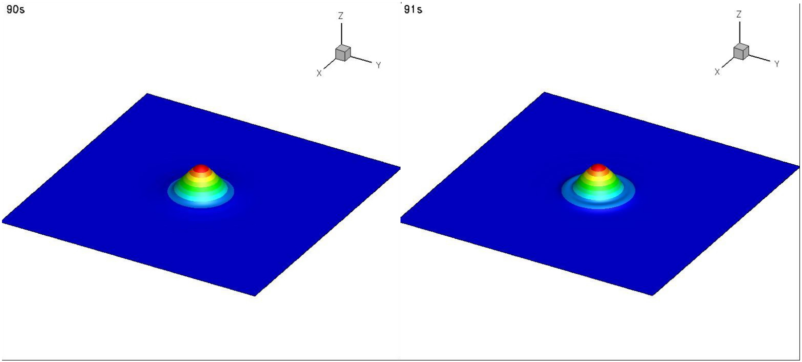


Figure: Evolution for the exploding $t = 90s$, $t = 91s$

Exploding/Erupting

- Envelopes begin to degenerate, going from a radially Gaussian shape to regions of its slopes that cave in

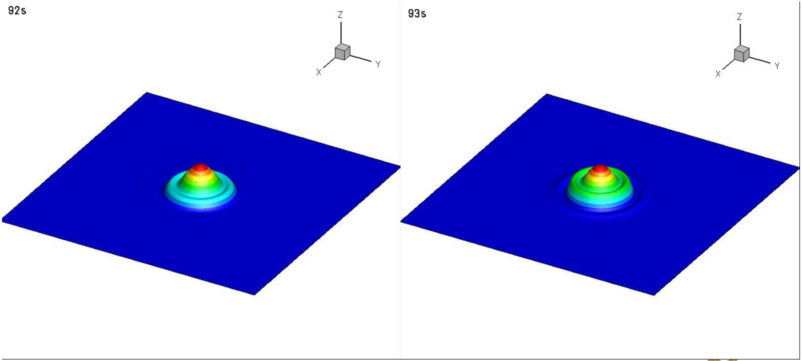


Figure: Evolution for the exploding $t = 92s$, $t = 93s$

Exploding/Erupting

- Then, soliton explodes intermittently, resulting in significant bursts of power above, but it recovers the initial shape after the explosion

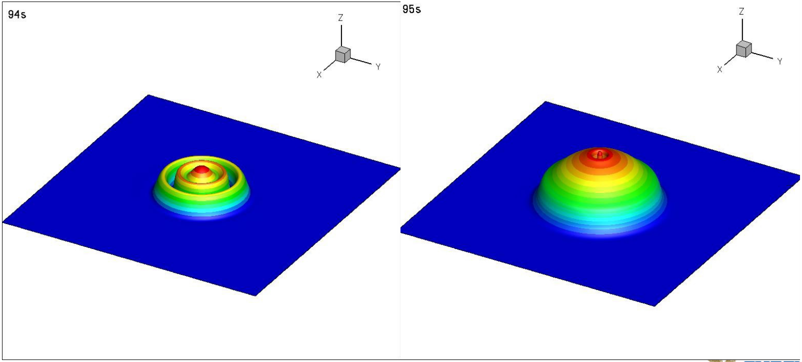


Figure: Evolution for the exploding $t = 94s$, $t = 95s$

Exploding/Erupting

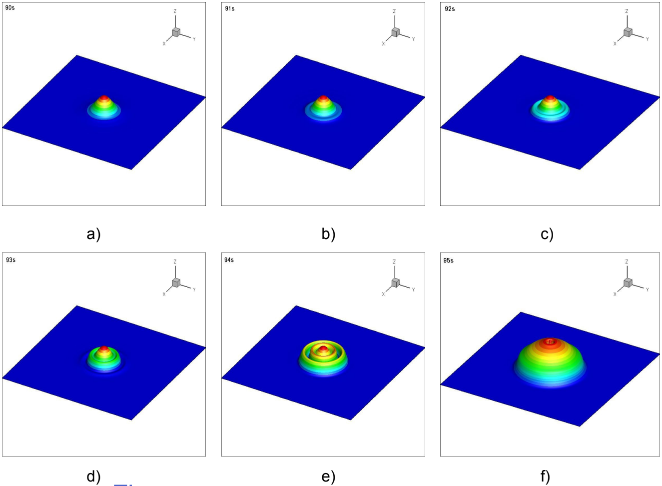
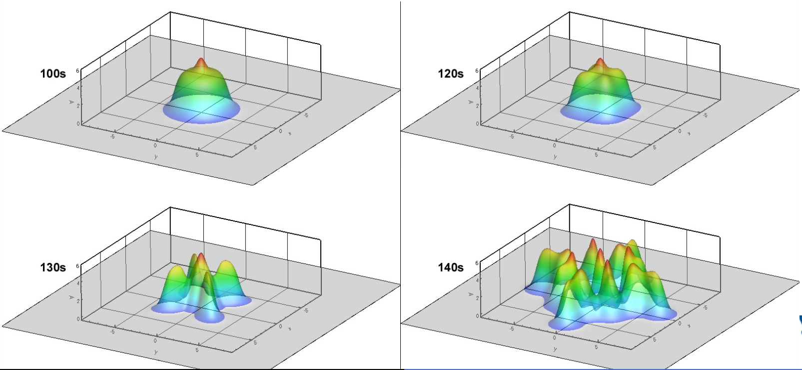


Figure: Evolution for the exploding soliton

Creeping

- Gaussian IC, $A_0 = 3.0$, and circular $\sigma_x = \sigma_y = 0.25$
- Creeping Soliton for 0-100 s
- Creeping Soliton for 100-200 s It changes its shape and shifts a finite distance periodically while remains confined to domain



Energy for Creeping

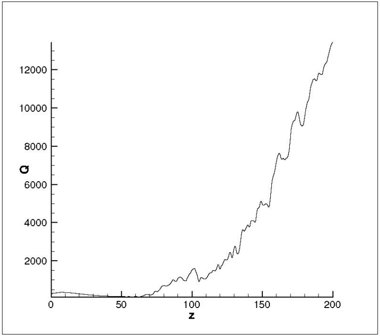


Figure: Energy for creeping soliton

3D Solitons

- Vary parameters for all classes of solitons
- Increase vorticity $m > 1$
- Study the stability regimes, transitions to instability, breaking, emerging a new class or non-existing (dissipating)
- Develop 3D numerical schemes, light bullets
- Soliton-soliton interaction

Acknowledgment

- Computations were performed on a Linux cluster (256 Intel Xeon 3.2GHz 1024 KB cache 4GB with Myrinet MX, GNU Linux) at ERAU

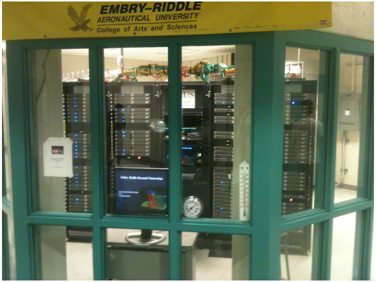


Figure: ZEUS cluster at ERAU

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