ON THE JACOBIAN GROUP FOR MOEBIUS LADDER AND PRISM GRAPHS

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CONTEXT

1. Notation and terminology.
2. Jacobian group of finite graph
3. Moebius ladder
4. Prism Graph
5. Connection between ML and PrG
6. Chebyshev polynomials
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Notation and terminology.

G is finite, connected multigraph without loops.

Let *V (G)* and *E(G)* be the sets of vertices and edges of G, respectively.

Denote by *Div(G)* a free Abelian group on *V (G)*.

Notation and terminology.

We think of elements of *Div(G)* as formal integer linear combinations of elements of *V (G).* Each element

$$D \in Div(G)$$

can be uniquely presented as

$$D = \sum_{x \in V(G)} D(x)\, x, \quad D(x) \in Z.$$

Notation and terminology.

The *degree function*

$$\deg : Div(G) \to Z$$

is defined by

$$\deg(D) = \sum_{x \in V(G)} D(x)$$

Denote by $Div^0(G)$

the subgroup of *Div(G)* consisting of divisors of degree zero.

Notation and terminology.

Let f be a *Z*-valued function on *V (G).* We define the divisor of f by the formula

$$div(f)=\sum_{x\in V(G)}\sum_{xy\in E(G)}(f(x)-f(y))\,x\,.$$

Notation and terminology.

The divisor *div(f)* can be naturally identified with the graph-theoretic Laplacian of f.

Divisors of the form div(f), where f is a Z-valued function on V (G), are called *principal divisors.*

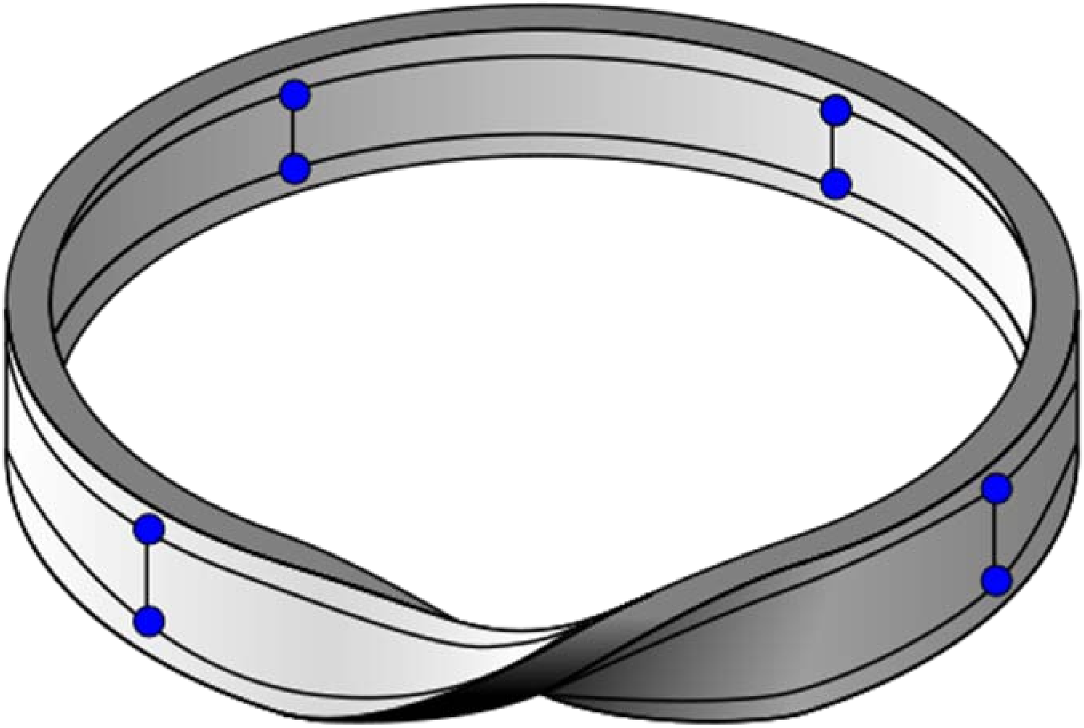
Denote by *Prin(G)* the group of principal divisors of *G.*

Jacobian group

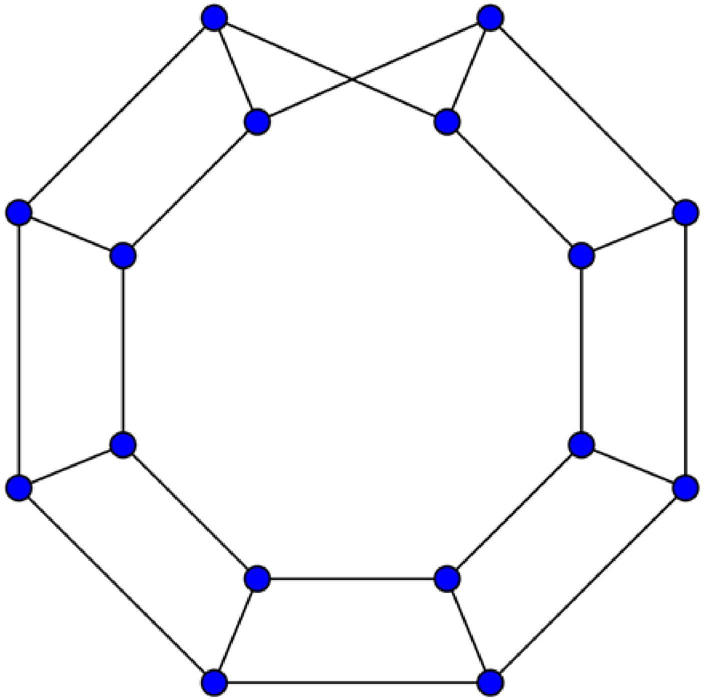
The Jacobian group (or Picard group) of G is defined to be quotient group

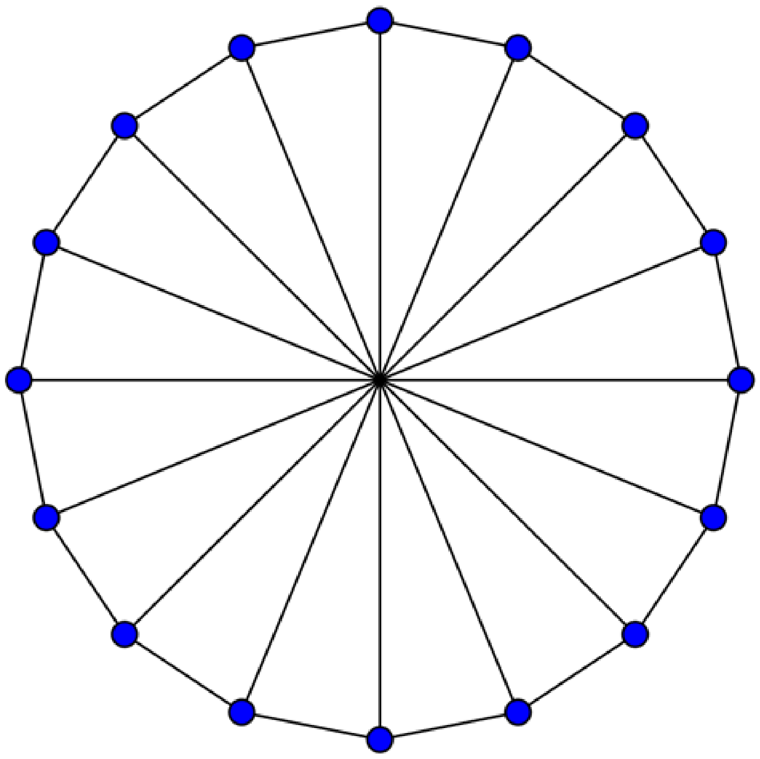
$$Jac(G) = {Div^0(G)} \big/ {Prin(G)} .$$

Moebius ladder

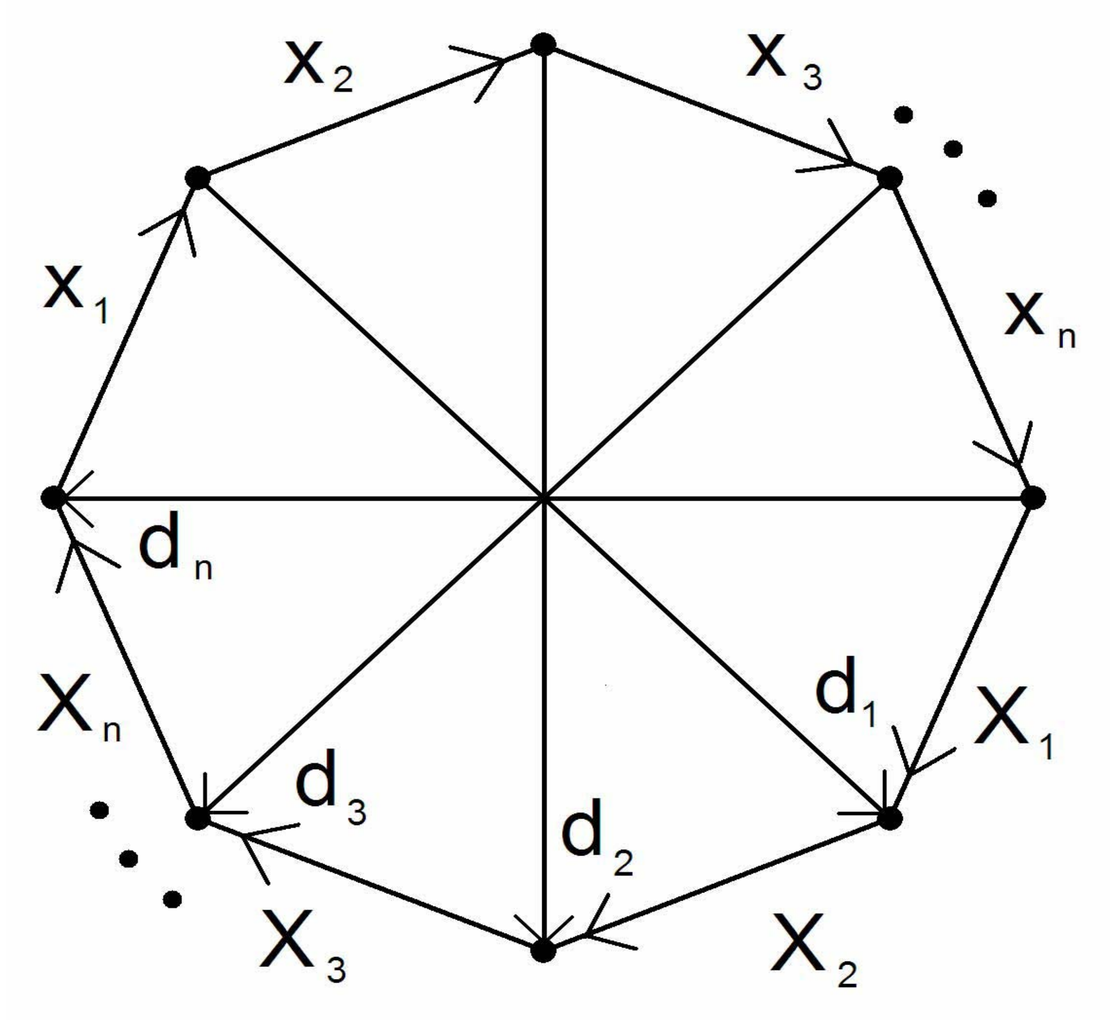


Moebius ladder M(8)

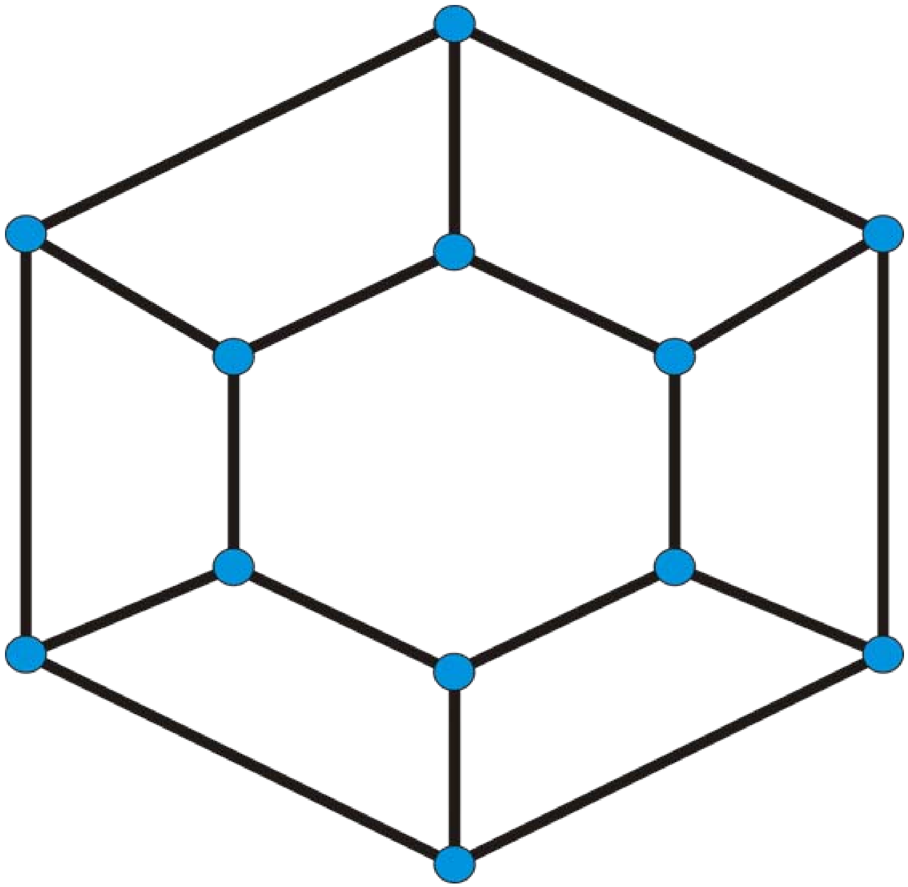




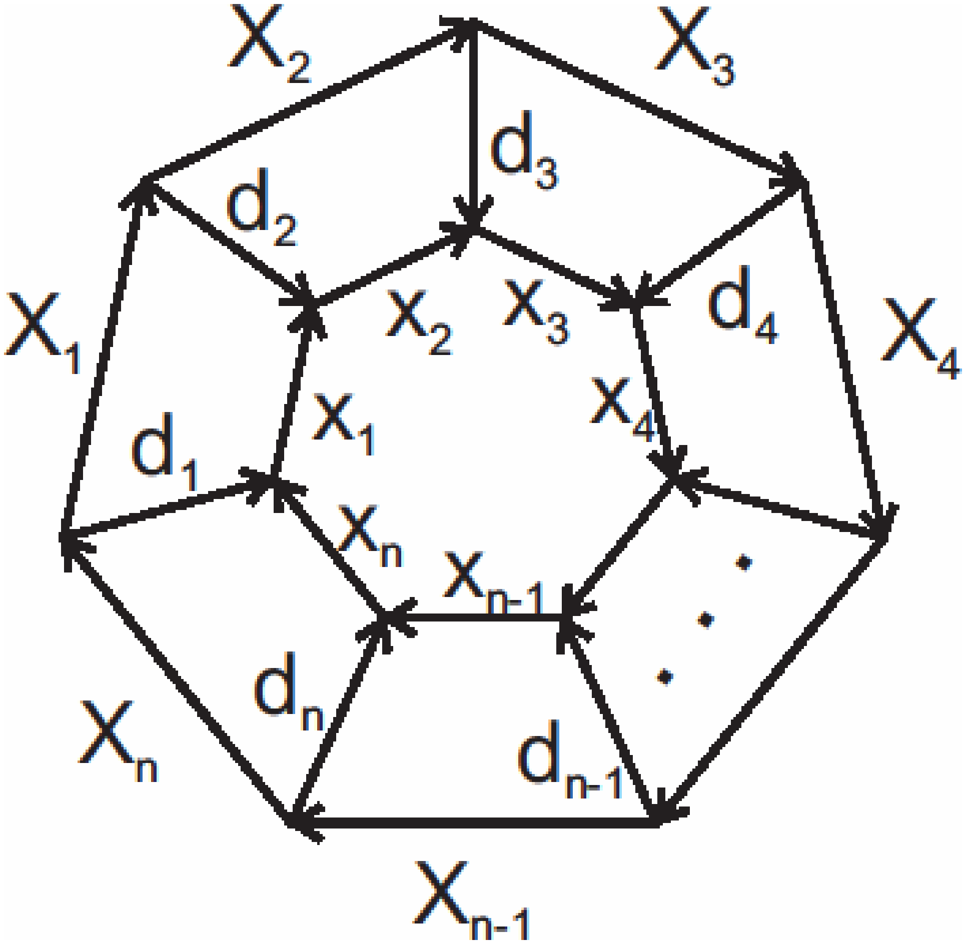
Moebius ladder



Prism graph Pr(6)



Prism graph



Connection between ML and PrG

Notice, that

Prism graph is a double cover of Moebius ladder.

It is discrete version of the statement :

The cylinder is a double cover of the Moebius band.

Chebyshev polynomials

$$T_n(x) = \cos(n \arccos x),$$

$$U_{n-1}(x) = \sin(n \arccos(x)) \big/ \sin(\arccos(x)).$$

$T_n(x)$, $U_{n-1}(x)$ are Chebyshev polynomials of the first and second kinds, respectively.

Chebyshev polynomials

$$T_n(x) = 2xT_{n-1}(x) - T_{n-2}(x),$$
$$T_0(x) = 1, \quad T_1(x) = x.$$

$$U_n(x) = 2x\,U_{n-1}(x) - U_{n-2}(x),$$
$$U_0(x) = 1, \quad U_1(x) = 2x.$$

Chebyshev polynomials

$$T_n(2) = \frac{(2+\sqrt{3})^n + (2-\sqrt{3})^n}{2},$$

$$U_{n-1}(2) = \frac{(2+\sqrt{3})^n - (2-\sqrt{3})^n}{2\sqrt{3}}.$$

$$T = T_n(2) + 1, \quad U = U_{n-1}(2),$$

$$L = T_n(2) - 1.$$

MAIN RESULTS

Theorem 1. The Jacobian of Moebius ladder M(n) has the following presentation

$$Jac(M(n)) = Z_{\frac{(n,T,U)}{(2,n)}} \oplus Z_{\frac{(T,nU)}{(n,T,U)}} \oplus Z_{\frac{(2,n)nT}{(T,nU)}},$$

where *(l, m, n) = GCD(l, m, n).*

MAIN RESULTS

Theorem 2. The Jacobian of the Prism graph *Pr(n)* has the following presentation

$$Jac(Pr(n)) = Z_{\frac{(n,L,U)}{(2,n)}} \oplus Z_{\frac{(L,nU)}{(n,L,U)}} \oplus Z_{\frac{(2,n)nL}{(L,nU)}},$$

where *(l, m, n) = GCD(l, m, n).*

REMARK

We note that the structure of the Jacobian groups Jac(M(n)) and Jac(Pr(n)) was independently investigated in [2] and [3] by completely different methods.

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Thanks

for

your attention!