Geometric Approach to Thermodynamics of Integrable Systems

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Integrable systems at finite T

- Black Holes in 3d and 4d
  – Eur. Phys. J. C 79 (2019) no.1, 71
  – Phys. Rev. D 99 (upcoming June 2019)
- Higher-order PU oscillators
  – Nucl. Phys. B 918 (2017) 317-336
- Strings and spin chains
  – Phys. Rev. D 96, 126004 (2017)
- Integrable Quadratic Hamiltonians
  – Procs. of XIX-GIQ-2017

Black hole thermodynamics

Black holes have entropy (Bekenstein-Hawking ’70s):

$$S = k_B \frac{A}{4L_p^2} + corrections \quad (1)$$

The first law of thermodynamics:

$$dM = TdS + \Omega dJ + \Phi dQ + \cdots = TdS + \Phi_i dQ^i = I_a dE^a \quad (2)$$

Black holes thermal stability (Davies ’80):

$$C = T\frac{\partial S}{\partial T}\begin{cases} > 0, & stable, \\ < 0, & radiating\ (unstable), \\ = 0, & phase\ transitions, \\ \to \infty, & phase\ transitions \end{cases} \quad (3)$$

Geometric approaches to the equilibrium space of black holes

The space of extensive parameters $\mathcal{E} = \{\Xi, E^a\}$ is called an equilibrium manifold if supplied with a proper metric structure.

- Hessian information metrics, "Geometric thermodynamics" (F. Weinhold 1975, G. Ruppeiner 1979)
- Legendre invariant metrics, "Geometrothermodynamics" (H. Quevedo 2006)
- Method of conjugate potentials, "New geometric thermodynamics" (B. Mirza, A. Mansoori 2014 & 2019)

Hessian metrics

Fluctuation theory (G. Ruppeiner '79):

$$S(E^a) = S_0 + EQL + \frac{\partial^2 S}{\partial E^a \partial E^b} dE^a dE^b + \cdots$$
$$= S_0 + EQL - g_{ab}(\vec{E}) dE^a dE^b \quad (4)$$

Ruppeiner information metric:

$$g_{ab}^{(R)} = -\frac{\partial^2 S}{\partial E^a \partial E^b} = -\mathrm{Hess} S(\vec{E}) \quad (5)$$

Weinhold information metric (F. Weinhold '75):

$$g_{ab}^{(W)} = \frac{\partial^2 M}{\partial E^a \partial E^b} = \mathrm{Hess} M(\vec{E}) \quad (6)$$

Scalar curvature and quantum gravity

1. The probability for fluctuating between macro states is proportional to the geodesic distance between them in $\mathcal{E}$.

2. The strength of interactions in the underlying QFT $\propto |R|$

3. The sign of R indicates the type of interactions in the underlying gauge theory (G. Ruppeiner '10):

$$R \begin{cases} > 0, & \textit{repulsive interactions}, \\ < 0, & \textit{attractive interactions}, \\ = 0, & \textit{free theory}, \\ \to \infty, & \textit{phase transitions} \end{cases} \tag{7}$$

4. Phase transitions = divergencies of R (F. Weinhold '75, G. Ruppeiner '79)

Legendre invariant metrics

- Consider $(2n+1)$ TD phase space $\mathcal{T}$ with coordinates $Z^A = (\Xi, I^a, E^a)$, $a = 1, \ldots, n$, where Ξ is a TD potential.
- Select on $\mathcal{T}$ a non-degenerate Legendre invariant metric $G = G(Z^A)$ and a Gibbs 1-form $\Theta(Z^A)$, namely

$$G^{GTD} = \Theta^2 + (\xi_{ab} E^a I^b)(\eta_{cd} dE^c dI^d), \quad \Theta = d\Xi - \delta_{ab} I^a dE^b,$$

  where δ_{ab} is the identity matrix, η_{ab} is the Minkowski metric, and ξ_{ab} is some constant tensor.
- Take the pullback $\phi^* : \mathcal{T} \to \mathcal{E}$ to find (H. Quevedo '17):

$$ds^2 = \left(\delta_{ac} \xi^{cb} E^a \frac{\partial \Xi}{\partial E^b} \right) \left(\eta^d_e \frac{\partial^2 \Xi}{\partial E^d \partial E^f} dE^e dE^f \right) \tag{8}$$

Conjugate thermodynamic potentials

For general black holes with $(m+1)$ TD variables, (S, Φ_i), and Enthalpy potential, $\bar{M} = M - \Phi_i Q_i$, one can define the metric (B. Mirza, A. Mansoori '19):

$$\hat{g} = \mathrm{blockdiag}\left(\frac{1}{T}\frac{\partial^2 M}{\partial S^2}, -\hat{G}\right), \tag{9}$$

where

$$G_{ij} = \frac{1}{T}\frac{\partial^2 M}{\partial Y^i \partial Y^j}, \quad Y^i = (Q_1, \ldots, Q_m) \tag{10}$$

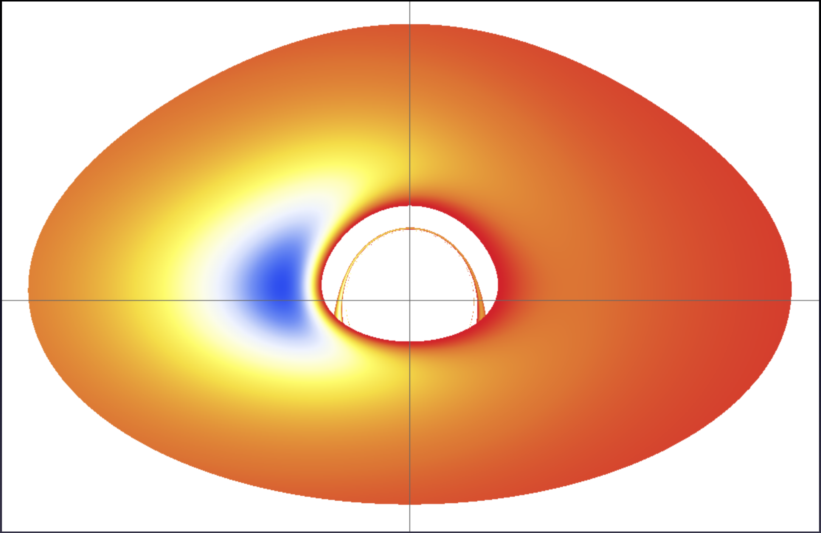
CFT_2/BH_3 duality

1. TIG for 3d $WAdS_3$ black hole solution in TMG dual to $WCFT_2$ with left and right central charges (H. Dimov, R. C. Rashkov, T. Vetsov '19):

$$T_c = \frac{1}{\pi(c_L + \sqrt{c_L c_R})} \tag{11}$$

2. TIG for 3d $SLifBH_3$ black hole solution in NMG dual to $UnknownCFT_2$ (K. Kolev, K. Staykov, T. Vetsov '19):

$$\mathcal{G}_{ij} = \frac{\partial^2 \psi}{\partial \lambda^i \partial \lambda^j} = \langle (X_i - \langle X_i \rangle)\,(X_j - \langle X_j \rangle) \rangle \,. \tag{12}$$



G. Gyulchev, P. Nedkova, T. Vetsov and S. Yazadjiev
[arXiv:1905.05273 [gr-qc]]

Summary and future R&D

- Thermodynamic information geometry is a set of geometric tools for investigating statistical thermal systems in equilibrium or non-equilibrium.
- Information Geometry – understanding how classical and quantum information can be encoded onto the degrees of freedom of any physical system.
- Future Research and Development: non-equilibrium physics, complexity, chaos, second variations, etc.

Credits

Thank You!

- In colaboration with R. C. Rashkov and H. Dimov:
  – H. Dimov, R. C. Rashkov, T. Vetsov, 1902.02433 [hep-th] (accepted in Phys. Rev. D)
- In colaboration with K. Kolev and K. Staykov:
  – K. Kolev, K. Staykov and T. Vetsov (in progress)

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$x^3 + y^3 + z^3 = 42?$
$x, y, z \in \mathbb{Z}$