MIC-KEPLER PROBLEM in classical mechanics – The **Hamiltonian system** (M, ω_μ, H_μ), where $M = T^*\dot{\mathbf{R}}^3$, $\dot{\mathbf{R}}^3 = \mathbf{R}^3 \setminus \{0\}$,

$$\omega_\mu = \sum_{i=1}^{3} dp_i \wedge dq_i - \frac{\mu}{2|q|^3} \sum_{i,j,k=1}^{3} \epsilon_{ijk} q_i dq_j \wedge dq_k$$

and

$$H_\mu = \frac{|p|^2}{2} - \frac{\alpha}{|q|} + \frac{\mu^2}{2|q|^2}, \qquad \alpha, \mu \in \mathbf{R}, \quad \alpha > 0,$$

which can be considered as a one-parameter deformation family of the standard Kepler problem (M, ω_0, H_0) with the remarkable property that it retains its high dynamical symmetries. Physically, the deformation parameter μ is interpreted as the magnetic charge of the particle at rest and measures the pitch of the cone on which trajectories lie. Its genuine mathematical interpretation is as a **cohomology** class of the symplectic structure. The global symmetry group of the problem is either SO(4), $E(3)$ or SO(3, 1), depending on whether the energy is negative, zero or positive. The motion satisfies Kepler's three laws (cf. **Kepler equation**).

The Hilbert spaces associated with the quantized problem carry almost all unitary irreducible representations of the respective covering groups, the only exception being the group SL(2, **C**), for which only the **principal series** representations arise. All this allows one to derive the spectrum and multiplicities of the bound states, as well differential cross sections of the scattering process and quantizations of the magnetic charge.

References

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MSC 1991: 58F05, 58F07, 70H33, 81R05, 81S10