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ABELIAN CONNECTION IN FEDOSOV DEFORMATION QUANTIZATION

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Abstract. General properties of an Abelian connection in Fedosov deformation quantization are investigated. The definition and the criterion of being a finite formal series for an Abelian connection are presented. Examples of finite and infinite Abelian connections are given.

1. Introduction

Deformation quantization of the phase space $\mathbb{R}^{2n}$ was invented in the middle of the previous century. Making use of the results obtained by Weyl [9], Wigner [10] and Groenewold [5] Moyal [7] presented quantum mechanics perceived as a statistical theory.

The first successful generalization of Moyal's results in case of a phase space different from $\mathbb{R}^{2n}$ appeared in 1977 when Bayen *et al.* [1] proposed an axiomatic version of the deformation quantization. In those articles quantum mechanics gained a new aspect – as a deformed version of the classical physics.

One of the realizations the quantization programme of Bayen *et al.* is the so called Fedosov deformation quantization [2, 3]. The Fedosov construction is algebraic and can be applied easily for example to solve the harmonic oscillator or to find momentum and position eigenvalues and Wigner eigenfunctions on a 2-D symplectic space with constant curvature tensor [4]. A great advantage of that method is the fact that computations may be done by computer programmes.

In Fedosov quantization we work with formal series. There is no general method to write these series in a compact form. Series of compact form appear for example when they contain finite number of terms. In that case the $*$-product of functions can be calculated exactly.

Fedosov deformation quantization is based on two recurrent equations. The first one is the formula defining an Abelian connection, the second – a relation in-