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RIGGINGS OF LOCALLY COMPACT ABELIAN GROUPS

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Abstract. We obtain a set of generalized eigenvectors that provides a generalized spectral decomposition for a given unitary representation of a commutative, locally compact topological group. These generalized eigenvectors are functionals belonging to the dual space of a rigging on the space of square integrable functions on the character group. These riggings are obtained through suitable spectral measure spaces.

1. Introduction

The purpose of the present paper is to take a first step towards a general formalism of unitary representations of groups and semigroups on rigged Hilbert spaces. To begin with, we want to introduce the theory corresponding to Abelian locally compact groups, leaving the more general nonabelian case as well as semigroups for a later work. We recall that a *rigged Hilbert space* or a rigging of a Hilbert space $\mathcal{H}$ is a triplet of the form

$$\mathbf{\Phi} \subset \mathcal{H} \subset \mathbf{\Phi}^{\times} \tag{1}$$

where $\mathbf{\Phi}$ is a locally convex space dense in $\mathcal{H}$ with a topology stronger than that inherited from $\mathcal{H}$ and $\mathbf{\Phi}^{\times}$ is the dual space of $\mathbf{\Phi}$. In this paper, we shall always assume that $\mathcal{H}$ is separable.

To each self adjoint operator A on $\mathcal{H}$, the von Neumann theorem [9] associates a *spectral measure space*. This is the quadruple $(\Lambda, \mathcal{A}, \mathcal{H}, P)$, where $\mathcal{H}$ is the Hilbert space on which A acts, $\Lambda = \sigma(A)$ is the spectrum of A, $\mathcal{A}$ is the family of Borel sets in Λ, and P is the projection valued measure on $\mathcal{A}$ determined by A through the von Neumann theorem. Obviously $\Lambda \subset \mathbb{R}$. A complete discussion on the relation between these concepts can be found in [3]. We say that the topological vector space $(\mathbf{\Phi}, \tau_{\mathbf{\Phi}})$ (vector space $\mathbf{\Phi}$ with the locally convex topology given by $\tau_{\mathbf{\Phi}}$) equips or rigs the spectral measure $(\Lambda, \mathcal{A}, \mathcal{H}, P)$ if the following conditions hold: