



## ABOUT THE DENSITIES FOR STRAIGHT LINES IN SEMI-RIEMANNIAN SPACES

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**Abstract.** We find some new expressions of density for straight lines and planes in euclidean spaces of dimension two and three. Also, we show the density for straight lines in surfaces which is expressed in terms of the coefficient of the first fundamental form. Finally, the density for tangent lines to a differentiable plane curve and for tangent plane of a differentiable surface are presented.

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### 1. Introduction

Since the Buffon's needle problem in the XVIII century until now, through the Crofton's formulas, many papers were written about different expressions of the density for straight lines and its application to integral formulas related to a convex set and further to geometric probability.

The applications to tomography, [1] and stereology show us that we are far from considering that the interest in these differential forms obtained under different hypothesis is over.

Even in [2] we can find the technical way to obtain them and the corresponding supported theory but, for instance, the density for one and two dimensional linear spaces in  $\mathbb{R}^3$  are not exhibit leaving many of integral formulas related to convex bodies as an open problems.

In this paper we want to show a most general expression of density for straight lines in semi-Riemannian spaces of dimension two and for lines and planes in  $\mathbb{R}^3$ . In particular, in the last sections we show density for tangent line to differential curves involving the curvature of the curve and for tangent planes to differential surfaces in terms of the Hessian of the surface.