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ON THE GEOMETRY OF BIHARMONIC SUBMANIFOLDS IN SASAKIAN SPACE FORMS

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Abstract. We classify all proper-biharmonic Legendre curves in a Sasakian space form and point out some of their geometric properties. Then we provide a method for constructing anti-invariant proper-biharmonic submanifolds in the Sasakian space forms. Finally, using the Boothby-Wang fibration, we determine all proper-biharmonic Hopf cylinders over homogeneous real hypersurfaces in complex projective spaces.

1. Introduction

As defined by Eells and Sampson in [14], *harmonic maps* $f : (M, g) \to (N, h)$ are the critical points of the *energy functional*

$$E(f) = \frac{1}{2}\int_M \|\mathrm{d}f\|^2 \, v_g$$

and they are solutions of the associated Euler-Lagrange equation

$$\tau(f) = \mathrm{tr}_g \nabla \mathrm{d}f = 0$$

where $\tau(f)$ is called the *tension field* of f. When f is an isometric immersion with mean curvature vector field H, then $\tau(f) = mH$ and f is harmonic if and only if it is minimal.

The *bienergy functional* (proposed also by Eells and Sampson in 1964, [14]) is defined by

$$E_2(f) = \frac{1}{2}\int_M \|\tau(f)\|^2 \, v_g.$$

The critical points of E_2 are called *biharmonic maps* and they are solutions of the Euler-Lagrange equation (derived by Jiang in 1986, [20]):

$$\tau_2(f) = -\Delta^f \tau(f) - \mathrm{tr}_g R^N(\mathrm{d}f, \tau(f))\mathrm{d}f = 0$$