JGSP **17** (2010) 35–43

JOURNAL OF
Geometry and Symmetry
in Physics

ON SURFACES OF LOW GENUS WHOSE TWISTOR LIFTS ARE HARMONIC SECTIONS

KAZUYUKI HASEGAWA

Communicated by Vasil V. Tsanov

Abstract. We determine surfaces of genus zero in four-dimensional hyperkähler manifolds whose twistor lifts are harmonic sections. As corollary, we prove that a surface of genus zero in four-dimensional Euclidean space is twistor holomorphic if its twistor lift is harmonic section.

1. Introduction

The twistor lifts play an important role and have been studied for oriented surfaces in oriented four-dimensional Riemannian manifolds by many researchers. For examples, in [2], it is proved that any oriented surface admits a conformal, superminimal immersion into the four-dimensional sphere. The twistor space is endowed with an almost complex structure, which is integrable when the base manifold is self-dual (see [1]). Surfaces with holomorphic twistor lifts are called twistor holomorphic surfaces (see [6]). The surfaces whose twistor lifts are harmonic sections have been studied in [8] and [9]. If the ambient spaces are self-dual Einstein, the twistor lifts of twistor holomorphic surfaces are harmonic sections. Note that recently surfaces whose twistor lifts are harmonic sections have been studied from the view point of the integrable systems in [3] and [11].

In this paper, we announce results for surfaces of genus zero in four-dimensional hyperkähler manifolds whose twistor lifts are harmonic sections. If certain conditions for the curvature tensors are satisfied and the twistor lift is a harmonic section, then the mean curvature vector field is a holomorphic section of the normal bundle with respect to the Koszul-Malgarange holomorphic structure (see [3], [8] and [9]). In general, holomorphic sections are not parallel. It is well-known that a compact constant mean curvature surface of genus zero immersed in $\mathbb{R}^3$ is totally umbilic, which is proved by Hopf in 1951. This theorem has been generalized in various settings, like higher codimension cases (see [4]). In these generalizations results, the mean curvature vector field is assumed to be parallel with respect