



## PROPERTIES OF BIHARMONIC SUBMANIFOLDS IN SPHERES

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**Abstract.** In the present paper we survey the most recent classification results for proper biharmonic submanifolds in unit Euclidean spheres. We also obtain some new results concerning geometric properties of proper biharmonic constant mean curvature submanifolds in spheres.

### 1. Introduction

*Biharmonic maps*  $\phi : (M, g) \rightarrow (N, h)$  between Riemannian manifolds are critical points of the *bienergy functional*

$$E_2(\phi) = \frac{1}{2} \int_M \|\tau(\phi)\|^2 v_g$$

where  $\tau(\phi) = \text{tr} \nabla d\phi$  is the tension field of  $\phi$  that vanishes for harmonic maps (see [17]). The Euler-Lagrange equation corresponding to  $E_2$  is given by the vanishing of the *bitension field*

$$\tau_2(\phi) = -J^\phi(\tau(\phi)) = -\Delta\tau(\phi) - \text{tr}R^N(d\phi, \tau(\phi))d\phi$$

where  $J^\phi$  is formally the Jacobi operator of  $\phi$  (see [24]). The operator  $J^\phi$  is linear, thus any harmonic map is biharmonic. We call *proper biharmonic* the non-harmonic biharmonic maps. Geometric and analytic properties of proper biharmonic maps were studied, for example, in [2, 25, 27].

The submanifolds with non-harmonic (non-minimal) biharmonic inclusion map are called *proper biharmonic submanifolds*. Initially encouraged by the non-existence results for proper biharmonic submanifolds in non-positively curved space forms (see, for example [8, 13, 16, 21]), the study of proper biharmonic submanifolds in spheres constitutes an important research direction in the theory of proper biharmonic submanifolds.

The present paper is organized as follows.