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MODULAR FORMS ON BALL QUOTIENTS OF NON-POSITIVE KODAIRA DIMENSION

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Abstract. The Baily-Borel compactification $\widehat{\mathbb{B}/\Gamma}$ of an arithmetic ball quotient admits projective embeddings by Γ-modular forms of sufficiently large weight. We are interested in the target and the rank of the projective map Φ, determined by Γ-modular forms of weight one. This paper concentrates on the finite H-Galois quotients $\mathbb{B}/\Gamma_H$ of a specific $\mathbb{B}/\Gamma_{-1}^{(6,8)}$, birational to an abelian surface A_{-1}. Any compactification of $\mathbb{B}/\Gamma_H$ has non-positive Kodaira dimension. The rational maps Φ^H of $\widehat{\mathbb{B}/\Gamma_H}$ are studied by means of the H-invariant abelian functions on A_{-1}.

The modular forms of sufficiently large weight are known to provide projective embeddings of the arithmetic quotients of the two-ball

$$\mathbb{B} = \{z = (z_1, z_2) \in \mathbb{C}^2;\, |z_1|^2 + |z_2|^2 < 1\} \simeq \mathrm{SU}(2,1)/\mathrm{S}(\mathrm{U}_2 \times \mathrm{U}_1).$$

The present work studies the projective maps, given by the modular forms of weight one on certain Baily-Borel compactifications $\widehat{\mathbb{B}/\Gamma_H}$ of Kodaira dimension $\kappa(\widehat{\mathbb{B}/\Gamma_H}) \leq 0$. More precisely, we start with a fixed smooth Picard modular surface $A'_{-1} = \left(\mathbb{B}/\Gamma_{-1}^{(6,8)}\right)'$ with abelian minimal model $A_{-1} = E_{-1} \times E_{-1}$, $E_{-1} = \mathbb{C}/\mathbb{Z} + \mathbb{Z}\mathrm{i}$. Any automorphism group of A'_{-1}, preserving the toroidal compactifying divisor $T' = \left(\mathbb{B}/\Gamma_{-1}^{(6,8)}\right)' \setminus \left(\mathbb{B}/\Gamma_{-1}^{(6,8)}\right)$ acts on A_{-1} and lifts to a ball lattice Γ_H, normalizing $\Gamma_{-1}^{(6,8)}$. The ball quotient compactification $A'_{-1}/H = \overline{\mathbb{B}/\Gamma_H}$ is birational to A_{-1}/H. We study the Γ_H-modular forms $[\Gamma_H, 1]$ of weight one by realizing them as H-invariants of $[\Gamma_{-1}^{(6,8)}, 1]$. That allows to transfer $[\Gamma_H, 1]$ to the H-invariant abelian functions, in order to determine $\dim_{\mathbb{C}}[\Gamma_H, 1]$ and the transcendence dimension of the graded $\mathbb{C}$-algebra, generated by $[\Gamma_H, 1]$. The last one is exactly the rank of the projective map $\Phi : \widehat{\mathbb{B}/\Gamma_H} \dashrightarrow \mathbb{P}([\Gamma_H, 1])$.