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GEOMETRY OF VORTICES AND DOMAIN WALLS

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Abstract. In this paper we consider relativistic models that contain in their spectra of solutions extended topological defects. We find the geometrical constrains that describe deformed vortices and domain walls of constant width. Analytical form of these solutions in co-moving coordinates is identical with analytical form of the appropriate static solutions in the laboratory Cartesian coordinates. The geometrical constrains presented here describe fully the shape and the evolution of the vortices and domain walls of constant width.

1. Introduction

In many branches of physics significant part of the physical phenomena are described by topological solitons of varied types. The most important, in the midst of topological defects, are extended topological configurations. There are two types of these configurations: vortices and domain walls.

The vortices appear in the description of many low energy systems. The most important examples of condensed matter systems that contain vortices are superconductors of second type where vortices have a form of magnetic flux tubes that puncture the bulk of the superconducting material [16]. In liquid crystals vortices have a form of optical fibres [8], [6]. Vortices of topological origin were also measured in superfluid Helium [11]. It seems that vortices may also play some role in the modern cosmology [22], [18], [14]. Finally, there is hypothesis that quarks in barions and mesons are connected by vortices that have a form of color flux tubes [7], [15], [8], [20].

On the other hand, domain walls are usually observed in ferromagnetic and ferroelectric materials.

The typical model that contain vortices is an Abelian Higgs model. This model describes a charged complex scalar field interacting with a vector potential (an electromagnetic potential)

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + (D^\mu\phi)^*(D_\mu\phi) + \frac{1}{4}\lambda(\phi\phi^* - a)^2 \tag{1}$$