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SMOOTH AND PEAKED SOLITONS OF THE CAMASSA-HOLM EQUATION AND APPLICATIONS

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Abstract. The relations between smooth and peaked soliton solutions are reviewed for the Camassa-Holm (CH) shallow water wave equation in one spatial dimension. The canonical Hamiltonian formulation of the CH equation in action-angle variables is expressed for solitons by using the scattering data for its associated isospectral eigenvalue problem, rephrased as a Riemann-Hilbert problem. The momentum map from the action-angle scattering variables $T^*(\mathbb{T}^N)$ to the flow momentum provides the Eulerian representation of the N-soliton solution of CH in terms of the scattering data and squared eigenfunctions of its isospectral eigenvalue problem. The dispersionless limit of the CH equation and its resulting peakon solutions are examined by using an asymptotic expansion in the dispersion parameter. The peakon solutions of the dispersionless CH equation in one dimension are shown to generalize in higher dimensions to peakon wave-front solutions of the EPDiff equation whose associated momentum is supported on smoothly embedded subspaces. The Eulerian representations of the singular solutions of both CH and EPDiff are given by the (cotangent-lift) momentum maps arising from the left action of the diffeomorphisms on smoothly embedded subspaces.

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