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ON SOLITON INTERACTIONS FOR THE HIERARCHY OF A GENERALISED HEISENBERG FERROMAGNETIC MODEL ON $\mathrm{SU}(3)/\mathrm{S}(\mathrm{U}(1)\times\mathrm{U}(2))$ SYMMETRIC SPACE

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Abstract. We consider an integrable hierarchy of nonlinear evolution equations (NLEE) related to linear bundle Lax operator L. The Lax representation is $\mathbb{Z}_2\times\mathbb{Z}_2$ reduced and can be naturally associated with the symmetric space $\mathrm{SU}(3)/\mathrm{S}(\mathrm{U}(1)\times\mathrm{U}(2))$. The simplest nontrivial equation in the hierarchy is a generalization of Heisenberg ferromagnetic model. We construct the N-soliton solutions for an arbitrary member of the hierarchy by using the Zakharov-Shabat dressing method with an appropriately chosen dressing factor. Two types of soliton solutions: quadruplet and doublet solitons are found. The one-soliton solutions of NLEEs with even and odd dispersion laws have different properties. In particular, the one-soliton solutions for NLEEs with even dispersion laws are *not* traveling waves while their velocities and amplitudes are time dependent. Calculating the asymptotics of the N-soliton solutions for $t\to\pm\infty$ we analyze the interactions of quadruplet solitons.

1. Introduction

The main object of present paper is the following coupled system of equations

$$\begin{aligned} &\mathrm{i}u_t + u_{xx} + (uu_x^* + vv_x^*)u_x + (uu_x^* + vv_x^*)_x u = 0\\ &\mathrm{i}v_t + v_{xx} + (uu_x^* + vv_x^*)v_x + (uu_x^* + vv_x^*)_x v = 0 \end{aligned} \tag{1}$$

where the smooth functions $u:\mathbb{R}^2\to\mathbb{C}$ and $v:\mathbb{R}^2\to\mathbb{C}$ satisfy the algebraic constraint $|u|^2+|v|^2=1$. The system (1) is a natural candidate to be a multicomponent generalisation of the classical Heisenberg ferromagnetic equation. It is well known [32] that the Heisenberg ferromagnetic model is integrable in the sense of inverse scattering method (ISM). It has a Lax pair related to the algebra $\mathfrak{su}(2)$. Since the time the complete integrability of HF equations was discovered, many attempts for its generalization have been made [20–22]. A well known method [10, 12, 24, 26–31] to obtain new integrable nonlinear evolution equations