



## STAR PRODUCT AND ITS APPLICATION TO THE MIC-KEPLER PROBLEM

TOMOYO KANAZAWA AND AKIRA YOSHIOKA

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**Abstract.** We show that the MIC-Kepler problem is simply solved via the phase-space formulation of non-relativistic quantum mechanics. The MIC-Kepler problem is the Hamiltonian system behind the hydrogen atom subjected to the influence of the Dirac's magnetic monopole field and the square inverse centrifugal potential force besides the Coulomb's potential force. We get the energy spectrum of the bound states explicitly and construct the Green's functions for  $E < 0$  by means of the Moyal product, which is one of the 'star' products denoted by ' $*$ '.

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### 1. Introduction

In 1978, Bayen *et al* [1] demonstrated that the quantum mechanics could be replaced by a "deformation" of the classical mechanics by introducing an associative algebra ( $*$ -product algebra) and the corresponding Lie algebra. The Moyal product and the associated Moyal bracket are the most familiar instance of them, which are directly connected with the definition of quantum commutator.

In this paper, using a star product, more precisely using the Moyal product, we calculate the energy spectrum and Green's functions of the MIC-Kepler problem in the Weyl-Wigner-Moyal (WWM) formalism, which furnishes an alternative formulation – historically, the latest – of quantum mechanics that is independent of the conventional Hilbert space and path integral approach.

The motion of the electron in the hydrogen atom is called quantum-mechanical Kepler problem. In 1970, McIntosh and Cisneros studied the above-mentioned dynamical system and treated the motion of an electron not only in the Coulomb's potential but also in both magnetic monopole field derived from a vector potential and a centrifugal potential proportional to the square of the pole strength, which is referred later on as the MIC-Kepler problem [7].

Our notation is as follows:  $m$  and  $k$  are positive constants which denote the mass and the charge of the electron. Moreover Planck's constant  $\hbar (> 0)$  appears which