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f-BIHARMONIC MAPS BETWEEN RIEMANNIAN MANIFOLDS

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Abstract. We show that if ψ is an f-biharmonic map from a compact Riemannian manifold into a Riemannian manifold with non-positive curvature satisfying a condition, then ψ is an f-harmonic map. We prove that if the f-tension field $\tau_f(\psi)$ of a map ψ of Riemannian manifolds is a Jacobi field and ϕ is a totally geodesic map of Riemannian manifolds, then $\tau_f(\phi \circ \psi)$ is a Jacobi field. We finally investigate the stress f-bienergy tensor, and relate the divergence of the stress f-bienergy of a map ψ of Riemannian manifolds with the Jacobi field of the $\tau_f(\psi)$ of the map.

1. Introduction

Harmonic maps between Riemannian manifolds were first established by Eells and Sampson in 1964. Afterwards, there are two reports and one survey paper by Eells and Lemaire [15–17] about the developments of harmonic maps up to 1988. Chiang, Ratto, Sun and Wolak also studied harmonic and biharmonic maps in [4–9]. f-harmonic maps which generalize harmonic maps, were first introduced by Lichnerowicz [25] in 1970, and were studied by Course [12, 13] recently. The f-harmonic maps relate to the equation of the motion of a continuous system of spins with inhomogeneous neighbor Heisenberg interaction in mathematical physics. Moreover, F-harmonic maps between Riemannian manifolds were first introduced by Ara [1, 2] in 1999, which could be considered as the special cases of f-harmonic maps.

Let $f : (M_1, g) \to (0, \infty)$ be a smooth function. By definition the f-biharmonic maps between Riemannian manifolds are the critical points of f-bienergy

$$E_2^f(\psi) = \frac{1}{2} \int_{M_1} f|\tau_f(\psi|^2 \mathrm{d}v$$

where $\mathrm{d}v$ the volume form determined by the metric g. The f-biharmonic maps between Riemannian manifolds which generalized biharmonic maps by Jiang [20, 21] in 1986, were first studied by Ouakkas, Nasri and Djaa [27] in 2010.

In section two, we describe the motivation, and review f-harmonic maps and their relationship with F-harmonic maps. In Theorem 3.1, we show that if ψ is an