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ON THE GENERALIZED f-BIHARMONIC MAPS AND STRESS f-BIENERGY TENSOR

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Abstract. In this paper, we investigate some properties for generalized f-harmonic and f-biharmonic maps between two Riemannian manifolds. In particular we present some new properties for the generalized stress f-energy tensor and the divergence of the generalized stress f-bienergy.

1. Introduction

Consider a smooth map $\varphi : M \longrightarrow N$ between Riemannian manifolds $M = (M^m, g)$ and $N = (N^n, h)$ and $f : M \times N \longrightarrow (0, +\infty)$ is a smooth positive function, then the f-energy functional of φ is defined by

$$E_f(\varphi) = \frac{1}{2}\int_M f(x, \varphi(x))\, |\mathrm{d}_x\varphi|^2\, v_g$$

(or over any compact subset $K \subset M$).

A map is called f-harmonic if it is a critical point of the $E_f(\varphi)$. In terms of Euler-Lagrange equation, φ is harmonic if the f-tension field of φ

$$\tau_f(\varphi) = f_\varphi \tau(\varphi) + \mathrm{d}\varphi(\mathrm{grad}^M f_\varphi) - e(\varphi)(\mathrm{grad}^N f) \circ \varphi.$$

The f-bienergy functional of φ is defined as

$$E_{2,f}(\varphi) = \frac{1}{2}\int_M |\tau_f(\varphi)|^2\, v_g.$$

A map is called f-biharmonic if it is a critical point of the f-bienergy functional.

The f-harmonic and f-biharmonic concept is a natural generalization of harmonic maps (Eells and Sampson [8]), and biharmonic maps (Jiang [9]).

In mathematical physics, f-harmonic maps, are related to the equations of the motion of a continuous system of spins (see [6]) and the gradient Ricci-soliton structure (see [12]).

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