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SOLVABLE AND/OR INTEGRABLE MANY-BODY MODELS ON A CIRCLE

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Abstract. Various many-body models are treated, which describe N points confined to move on a plane circle. Their Newtonian equations of motion ("accelerations equal forces") are *integrable*, i. e. they allow the *explicit* exhibition of N *constants of motion* in terms of the dependent variables and their time-derivatives. Some of these models are moreover *solvable* by purely algebraic operations, by (explicitly performable) quadratures and, finally, by functional inversions. The techniques to manufacture these models are not new; some of these models are themselves new; others are reinterpretations of known models.

1. Introduction

The investigation of the time evolution of an arbitrary number N of point-particles the dynamics of which is determined by Newtonian equations of motion ("accelerations equal forces") is of course a fundamental topic in physics and mathematics. The identification in this context of models *amenable to exact treatments* is a major area of research in mathematical physics and applied mathematics, having a centuries-old history and having been boosted by developments in the last few decades, which also impacted several areas of physics beyond mechanics and many fields of pure mathematics. An interesting related development which is now becoming of interest is the study of such models in which the motion is restricted to lie on an *a priori* prescribed manifold: see for instance [1, 5, 6, 8]. In this paper we make some initial, simple steps in this direction by focussing on various many-body models describing the evolution of N points whose positions on a plane are characterized by N *unit* two-vectors, thereby forcing their motion to be confined to *a circle of unit radius centered at the origin*. All these models are characterized by *Newtonian* equations of motion: accelerations equal forces, which in these models are of *one-body*, *two-body* or, in some cases, *many-body* type, and might depend on the velocities of the moving particles in addition to their positions. All these models are *autonomous*: their equations of motion are time-independent. They are *all amenable to exact treatments*: in particular they *all*