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A DECOUPLED SOLUTION TO THE GENERALIZED EULER DECOMPOSITION PROBLEM IN $\mathbb{R}^3$ AND $\mathbb{R}^{2,1}$

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Abstract. In this article we suggest a new method, partially based on earlier works of Wohlhart [15], Mladenova and Mladenov [11], Brezov et al [3], that resolves the generalized Euler decomposition problem (about arbitrary axes) using a system of quadratic equations. The main contribution made here is that we manage to decouple this system and express the solutions independently in a compact covariant form. We apply the same technique to the Lorentz group in $2+1$ dimensions and discuss certain complications related to the presence of isotropic directions in $\mathbb{R}^{2,1}$.

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