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# ON THE CONSTRUCTION OF RECURSION OPERATORS FOR THE KERR-NEWMAN AND FRLW METRICS

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**Abstract.** We consider complete integrability of the Hamiltonian of the geodesic flow of two particular solutions, the Kerr-Newman and the FRLW metrics of the Einstein equations in the sense of Liouville. We construct recursion operators using first integrals, and then obtain constants of motion of the geodesic flows by using the recursion operators.

*MSC*: 37J35, 53D25
*Keywords*: recursion operator, geodesic flows, Hamiltonian systems, completely integrable systems.

## 1. Introduction

We consider two particular solutions of the Einstein equations, the Kerr-Newman and the FRLW metrics. The Kerr-Newman metric is a metric of space-time symmetry axis representing the black hole that was charged to rotation (see e.g. [6]). The FRLW metric stands for the Friedmann-Lemaître-Robertson-Walker metric, which is widely used as a first approximation of the expanding universe model (see e.g., [8]). These metrics are well-known as the exact solutions of the Einstein equations.

In [10], we get complete integrability of the Hamiltonian of the geodesic flows of four solutions of the Einstein equations: Schwarzschild, Reissner-Nordström, Kerr and Kerr-Newman metrics. In this paper, we show the Hamiltonian function of the geodesic flow of the Kerr-Newman metric and the FRLW metric are system of separation of variables, and then we get complete integrability of the Hamiltonian of the geodesic flow of the Kerr-Newman metric and the FRLW metric in the sense of Liouville, respectively.

In [1] and [2] the authors proposed a new characterization of integrable systems, which is called a recursion operator. A recursion operator is a $(1,1)$-tensor field which satisfies the conditions: $1)$ Lie derivative is zero under a dynamical vector