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ON REFLECTIONS AND ROTATIONS IN MINKOWSKI 3-SPACE

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Abstract. In this paper, we investigate the reflections in Minkowski three-space by three different approaches. Firstly, we define Lorentzian reflections with Lorentzian inner product. Then, we examine Lorentzian reflections in view of Lorentzian Householder matrices. Finally, we use pure split quaternions to derive Lorentzian reflections. For each case, we find the matrix representation of Lorentzian reflections and characterize the plane of reflection by using this matrix representation. Moreover, we prove that any Lorentzian orthogonal transformation can be represented by the composition of at most three reflections.

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1. Introduction

In the Euclidean space, a reflection is an isometry with a hyperplane as a set of fixed points. This set is called the axis (in dimension two) or plane (in dimension three) of reflection. The image of a figure by a reflection is its mirror image in the axis or plane of reflection. A reflection along a subspace in $\mathbb{R}^n$ maintains the perpendicular distance from this subspace and its orthogonal complement. And orthogonal complement of this subspace separates the initial and reflected vectors. Thus the components of the initial vector orthogonal to the subspace along which the reflection occurs remain unchanged.

A reflection in $\mathbb{R}^n$ can be represented by an $n \times n$ symmetric orthogonal matrix with determinant -1 and eigenvalues (1, 1, 1, ... 1, -1). The product of two such matrices is a special orthogonal matrix which represents a rotation. Every rotation is the result of reflecting in an even number of reflections in hyperplanes through the origin. Thus, the reflections generate orthogonal groups and this result is known as the Cartan–Dieudonné theorem. In [3], answers of the following two frequently asked questions are handled:

Question 1. *In how many ways can an orthogonal transformation in an n-dimensional Euclidean space be decomposed?*

Question 2. *What are the simple reflections that determine a given orthogonal*

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