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THE ADIABATIC LIMIT FOR MULTIDIMENSIONAL HAMILTONIAN SYSTEMS

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Abstract. We study some properties of multidimensional Hamiltonian systems in the adiabatic limit. Using the properties of the Poincaré-Cartan invariant we show that in the integrable case conservation of action requires conditions on the frequencies together with conservation of the product of energy and period. In the ergodic case the most general conserved quantity is not volume but rather symplectic capacity; we prove that even in this case there are periodic orbits whose actions are conserved.

1. Introduction

Our purpose is to study some properties of Hamiltonian dynamics in the adiabatic limit for systems with an arbitrary number of degrees of freedom under certain conditions on the initial and final frequencies.

Put expeditiously, the adiabatic limit is the limit of slow change of some time-dependent parameters. For instance, the adiabatic limit of the motion of a pendulum with variable length and/or mass is the (ideal) motion of this pendulum when the rate of change of the frequency becomes "infinitely small". The adiabatic limit does usually not coincide with the limit of the dynamics obtained by "freezing" the parameter (if the parameters are kept constant, energy is conserved, while it is usually not in the adiabatic limit, cf. the pendulum). While the adiabatic invariance of Hamiltonian systems with one degree of freedom is well understood (see for instance Arnold's paper [2] for a thorough discussion), the case of multidimensional systems is far from being understood. This is due to the fact that the traditional approaches to the study of adiabatic invariance make use, at one moment or another, of an averaging procedure (i.e. a first order perturbation calculation), and such perturbation methods usually fail when there are several degrees of freedom. For a lucid discussion of the difficulties which appear