BOOK REVIEW

A modern mathematician, or mathematical physicist, could hardly imagine the
contemporary mathematics and mathematical physics without the concept of sym-
metry and associated techniques, formulated and used mostly in the modern dif-
ferential geometry language: this is what is known as Lie group actions on man-
ifolds. From physical point of view this is, of course, understandable, since the
real world consists of physical objects, and physical objects show two kind of
properties: intrinsic (proper, identifying) and behavioral (kinematical). The iden-
tifying properties of an object do not change during its time-evolution, while the
behavioral properties describe the admissible changes during evolution. So, this
"change-conservation" consistent nature of the physical objects finds its mathe-
matical adequacy in building mathematically realistic dynamical systems, such
that their dynamical equations are consistent with corresponding (i.e., physically
understandable) invariants. The mathematical tool for finding these invariants is,
namely, the concept of "invariance with respect to some Lie group action" on the
variables chosen. The local version of this approach, roughly speaking, consists
in finding well defined quantities (usually these are some combinations of proper
and kinematical characteristics of the system studied) having zero Lie derivatives
along the evolution defining objects, which in most cases are given by vector fields
on a manifold. As it is well known, having such conservative quantities at hand
simplifies the further study of the system, in fact, a corresponding reduction of
the independent degrees of freedom is possible. Moreover, in some cases just
knowing the conservative quantities of the system, not the detailed kinematics, is
of interest. For example, from physical point of view, having true expression for
the energy-momentum of the system allows to analyze successfully many aspects
of its future development. In symplectic, or Hamiltonian, mechanics the energy-
momentum is given by the corresponding "momentum map", which, in view of
the physical significance of the energy-momentum as universal and conservative

quantity, determines the importance of studying these momentum maps.

The subject of the book under review is, namely, a rigourous mathematical study of this concept in the comparatively well known Hamiltonian case, as well as, in the Poisson manifold case, and some its possible useful generalizations. The book consists of *Introduction*, eleven *Chapters*, *Bibliography* and *Index*. From one side, this is a good introductory text in symplectic geometry and Hamiltonian mechanics, and from the other side this is an excellent introduction to modern research in the field. Distinguished features of the book are its clarity, completeness, and due respect to the reader: everything is clearly explained, the proofs are easily understood, the examples are well chosen and worked out. The first two chapters have mainly introductory character and are devoted to smooth structures (we notice the stratified spaces in *Section 1.5*, Lie group actions (the concept of G-space introduced and briefly studied), while the third chapter introduces the more recent concepts of "pseudogroups" and "groupoids". The study of the momentum map begins in *Chapter 4*, it is illustrated with many examples. Some generalizations of the momentum map are considered in *Chapter 5*, these are "the cylinder valued momentum maps" and the "optimal momentum map". The optimal momentum map is closely related to the reduction procedure, which is fully represented in *Chapter 6*. A more detailed studies of the reduction procedure , as well as of the Poisson reduction, are given in *Chapters 7-10*. The book ends with *Chapter 11* devoted to the "notion of duality polarity in Poisson geometry". This is a more specified subject serving to introduce the reader into one of the modern directions of development inside symplectic geometry dealing with dual pairs, polarity, non-commutative integrable systems, and other important structures and properties related mainly to Poisson manifolds.

The book is written entirely in the coordinate-free language of modern differential geometry. So, a careful reading of the book creates the ability to think in invariant terms, which helps seriously in doing research in mathematics and mathematical physics. Moreover, getting knowledge of the material included opens new visions and perspectives also in studying infinite dimensional Hamiltonian systems as well. It should be highly recommended to all young researchers in the field of symplectic geometry and Hamiltonian mechanics.

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