



TAUB-NUT AS BERTRAND SPACETIME WITH MAGNETIC FIELDS

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Abstract. Based on symmetries Taub-NUT shares with Bertrand spacetime, we cast it as the latter with magnetic fields. Its nature as a Bianchi-IX gravitational instanton and other related geometrical properties are reviewed. We provide an easy derivation and comparison between the spatial Killing-Yano tensors deduced from first-integrals and the corresponding hyperkähler structures and finally verify the existence of a graded Lie-algebra structure via Schouten-Nijenhuis brackets.

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