



ON MATRIX REPRESENTATIONS OF GEOMETRIC (CLIFFORD) ALGEBRAS

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Abstract. Representations of geometric (Clifford) algebras with real square matrices are reviewed by providing the general theorem as well as examples of lowest dimensions. New definitions for isometry and norm are proposed. Direct and indirect isometries are identified respectively with automorphisms and antiautomorphisms of the geometric algebra, while the norm of every element is defined as the n^{th} -root of the absolute value of the determinant of its matrix representation of order n . It is deduced in which geometric algebras direct isometries are inner automorphisms (similarity transformations of matrices). Indirect isometries need reversion too. Finally, the most common isometries are reviewed in order to write them in this way.

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Contents

1	Introduction	2
2	Fundamentals of Matrix Representations	3
3	Representations of Geometric Algebras with Real Square Matrices	4
4	Representations of Lower-Dimensional Geometric Algebras	5
4.1	Complex Numbers ($\mathbb{C} = Cl_{0,1}$)	6
4.2	Hyperbolic Numbers ($\mathcal{H} = Cl_{1,0}$)	6
4.3	The Geometric Algebra of the Euclidean Plane $Cl_{2,0}$	6
4.4	The Geometric Algebra of the Hyperbolic Plane ($Cl_{1,1}$)	7
4.5	Quaternions ($\mathbb{H} = Cl_{0,2}$)	8
4.6	The Geometric Algebra of the Three-Dimensional Euclidean Space ($Cl_{3,0}$)	9
4.7	Pauli Matrices	10
4.8	Dirac Matrices and Spacetime Geometric Algebra	11
5	Representation of Geometric Algebras of any Dimension	13
5.1	Isomorphisms of Clifford Algebras	14