JGSP **44** (2017) 1–11

JOURNAL OF
Geometry and Symmetry
in Physics
ISSN 1312-5192

ON COMPLEX HOMOGENEOUS SPACE OF VECTORS WITH CONSTRAINTS

EMILIJA CELAKOSKA, ELENA HADZIEVA AND VESNA CELAKOSKA-JORDANOVA

Communicated by Ivaïlo Mladenov

Abstract. A homogeneous space $\mathcal{V}$ of complex constrained vectors in $\mathbb{C}^3$, representing complex velocities is introduced. The corresponding representation of the complex special orthogonal group of transformations acting on $\mathcal{V}$ is also examined. The requirement for real vector magnitudes is addressed by imposing orthogonality between the real and the imaginary parts of vectors and use of the non-conjugate scalar product. We present the orthogonal transformations acting on $\mathcal{V}$ in terms of the polar decomposition of complex orthogonal matrices. The group link problem and the homogeneity of the space $\mathcal{V}$ are also discussed. Finally, we briefly consider the convenience of the space $\mathcal{V}$ in theoretical calculations.

MSC: 51F25, 32M05, 20C35, 83D05
Keywords: Boost, complex orthogonal group, group link, group representation

1. Introduction

It is a common knowledge in the Lie group theory that the complex rotation group $\mathrm{SO}(3,\mathbb{C})$ is isomorphic to the restricted Lorentz group $\mathrm{SO}^+(1,3)$. Due to this isomorphism, the group $\mathrm{SO}(3,\mathbb{C})$ is sometimes an alternative choice to represent particular aspects of some physical theories ordinarily expressed in terms of $\mathrm{SO}^+(1,3)$. In regard to physical applications, it is important to outline some of the useful properties of the complex space $\mathbb{C}^3$ compared to commonly used four-dimensional real space: it supports the use of vector product, the increased number of vector components can be related to additional physical quantities and it offers a possibility to discern some commonly intertwined physical concepts by employing the real/imaginary separation. As examples of using $\mathrm{SO}(3,\mathbb{C})$ in physical applications one can see [1], [2], [10], [14] among others. It is noteworthy to remark that the representation of the group SO(3,$\mathbb{C}$) therein is adapted to the standard unconstrained vectors in $\mathbb{C}^3$. When the $\mathrm{SO}(3,\mathbb{C})$ transformation is parameterized by a vector of velocity, it is commonly taken to be a three-dimensional vector $\vec{v}$ with real components. Even when the vector of velocity is actually a complex vector

doi: 10.7546/jgsp-44-2017-1-11