



n-CHARACTERISTIC VECTOR FIELDS OF CONTACT MANIFOLDS

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Abstract. In present paper we define and study *n*-characteristic vector fields. We present definition of Tanaka-Webster connection, then use it for studying the behavior of *n*-characteristic vector fields. Also we show some results about of these vector fields by Tanaka-Webster connection.

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1. Introduction

The main goal of this paper is to study a special type of vector fields. These vector fields are defined in contact metric manifolds and called *n*-characteristic vector fields or briefly *n*-char vector fields. All of them are commutate with characteristic vector field and the bracket of both *n*-char vector fields is multiple of characteristic vector field and it is proved that the bracket of *n*-char vector fields commutate with other components of tangent bundle. It has been shown if tangent space of each contact metric manifold contained a *n*-char vector field, then characteristic vector is commutate with all vector fields. The *Tanaka-Webster connection* [3] first time defined by Shukichi Tanno for contact manifold. The study of *n*-char vector fields with *Tanaka-Webster connection* resulted in interesting results.

2. Preliminaries

Let *M* be an almost contact manifold, i.e., it is a $(2m + 1)$ -dimensional smooth manifold with an almost contact structure (φ, ξ, η) consisting of an endomorphism φ of the tangent bundle, a vector field ξ , its dual one-form η as well as *M* is equipped with a Riemannian metric *g*, so that the following relations are valid

$$\varphi\xi = 0, \quad \varphi^2 = -\text{Id} + \eta \otimes \xi, \quad \eta\xi = 1 \tag{1}$$

$$g(\varphi X, \varphi Y) = g(X, Y) - \eta(X)\eta(Y) \tag{2}$$