



COMPOSITION ALGEBRAS, EXCEPTIONAL JORDAN ALGEBRA AND RELATED GROUPS*

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Communicated by Ivailo M. Mladenov

Abstract. Normed division rings are reviewed in the more general framework of composition algebras that include the split (indefinite metric) case. The Jordan - von Neumann - Wigner classification of finite dimensional Jordan algebras is outlined with special attention to the 27 dimensional exceptional Jordan algebra \mathfrak{J} . The automorphism group F_4 of \mathfrak{J} and its maximal Borel-de Siebenthal subgroups $\frac{SU(3) \times SU(3)}{\mathbb{Z}_3}$ and $Spin(9)$ are studied in some detail with an eye to possible applications to the fundamental fermions in the Standard Model of particle physics.

MSC: 20G41, 17C40, 117A75, 7C60, 17A35, 22D20, 22E15, 22E20, 22E46

Keywords: Exceptional Lie groups, Jordan algebra, octonions, standard model

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*Dedicated to the memory of Professor Vasil V. Tsanov 1948-2017.