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INDEFINITE EISENSTEIN LATTICES: A MODERN BALL-RENDEVOUS WITH POINCARÉ, PICARD, HECKE, SHIMURA, MUMFORD, DELIGNE AND HIRZEBRUCH*

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Abstract. In [21] we have counted indefinite metrics (two-dimensional, integrally defined, over Gauss numbers) with a fixed norm (discriminant). We would like to call them also *indefinite class numbers*. In this article we change from Gauss to Eisenstein numbers. We have to work on the complex two-dimensional unit ball, an Eisenstein lattice on it and the quotient surface. It turns out that the compactified quotient is the complex plane $\mathbb{P}^2$. In the first part we present a new proof of this fact. In the second part we construct explicitly a Heegner series with the help of Legendre-symbol coefficients. They can be interpreted as "indefinite class numbers" we look for. Geometrically they appear also as number of plane curves with (normed) Eisenstein disc uniformization.

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Contents

*Dedicated to the memory of Professor Vasil V. Tsanov 1948-2017.

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