



UNSTEADY ROTO-TRANSLATIONAL VISCOUS FLOW: ANALYTICAL SOLUTION TO NAVIER-STOKES EQUATIONS IN CYLINDRICAL GEOMETRY

ALESSIO BOCCI, GIOVANNI MINGARI SCARPELLO AND DANIELE RITELLI

Communicated by Ivařilo M. Mladenov

Abstract. We study the unsteady viscous flow of an incompressible, isothermal (Newtonian) fluid whose motion is induced by the sudden swirling of a cylindrical wall and is also starting with an axial velocity component. Basic physical assumptions are that the pressure axial gradient keeps its hydrostatic value and the radial velocity is zero. In such a way the Navier-Stokes PDEs become uncoupled and can be solved separately. Accordingly, we provide analytic solutions to the unsteady speed components, i.e., the axial $v_z(r, t)$ and the circumferential $v_\theta(r, t)$, by means of expansions of Fourier-Bessel type under time damping. We also find: the surfaces of dynamical equilibrium, the wall shear stress during time and the Stokes streamlines.

MSC: 35Q30, 33C10

Keywords: Bessel functions, Navier-Stokes equations, unsteady flow

Contents

1	Introduction	2
1.1	On the Navier-Stokes Equations	2
1.2	Outline of Some Contemporary Literature	3
1.3	Aim of the Work	4
2	The Axial Sub-Problem	6
2.1	Axial Sub-Problem Outline	6
3	The Circumferential Sub-Problem	8
3.1	Circumferential Sub-Problem Outline	9
3.2	Integration of Equation (11)	10
3.3	A Profile Analysis of v_θ	12
3.4	The Pressure Field	13
3.5	The Liquid Free Surface $z(r, t)$	14