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CENTRALIZER OF REEB VECTOR FIELD IN CONTACT LIE GROUPS

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Abstract. We consider the centralizer of Reeb vector field of a contact Lie group with a left invariant Riemannian metric while contact structure is left invariant. Then we decompose the Lie algebra of this Lie groups to centralizer of Reeb vector field and its orthogonal complement and using this decomposition the contact Lie group is investigated. Furthermore, in last section a special automorphism is defined and studied which it keeps the contact form.

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1. Introduction and Preliminaries

Lie groups play an important role in many areas of mathematics and physics, special in quantum mechanics. The Lie groups contain two main aspects of mathematics, Algebra and Geometry, therefore these two topics can be linked in the Lie groups. Based on this possibility we want to study differential geometry of a contact metric Lie group's properties. We commence with basic definitions and concepts of Lie group and its contact structure.

Let G be a simply connected Lie group equipped by Riemannian left invariant metric $\langle\,,\rangle$ and $T_eG = \mathfrak{g}$ is Lie algebra of G, e is identity element of G. One would expect to find some properties that are similar to those in flat Euclidean space, which in this paper one can regard as a simply connected, abelian Lie group of translations with a canonical left invariant metric. Such features do exist, but other geometric features of $\{G, \langle\,,\rangle\}$ are foreign to Euclidean geometry. A Lie group H of a Lie group G is a subgroup which is also a submanifold and $\mathfrak{h}$ is Lie subalgebra of $\mathfrak{g}$. For all $X, Y \in \mathfrak{g}$, we have

$$[X, Y] = \nabla_X Y - \nabla_Y X$$

where ∇ is covariant derivative. Assume G is an odd dimensional Lie group with a left invariant metric $\langle\,,\rangle$, then G is said to be an almost contact metric Lie group