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A COMPLEX STRUCTURE ON THE MODULI SPACE OF RIGGED RIEMANN SURFACES

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Abstract. The study of Riemann surfaces with parametrized boundary components was initiated in conformal field theory (CFT). Motivated by general principles from Teichmüller theory, and applications to the construction of CFT from vertex operator algebras, we generalize the parametrizations to quasisymmetric maps. For a precise mathematical definition of CFT (in the sense of G. Segal), it is necessary that the moduli space of these Riemann surfaces be a complex manifold, and the sewing operation is holomorphic. We report on the recent proofs of these results by the authors.

1. Introduction

The results described in this paper are proved in detail in [19], which also contains an extensive introduction. As well as giving an overview of some of those results, we expand on certain conceptually important points. In particular, we explain why the use of quasisymmetric boundary parametrizations, in the geometric framework for conformal field theory, is both natural and necessary from the point of view of Teichmüller theory.

1.1. Description of Problem and Results

Let Σ^B be a bordered Riemann surface of type (g, n) with $n \geq 1$, i.e., a Riemann surface of genus g bounded by n closed curves $\partial_i \Sigma^B$, $i = 1, \ldots, n$, which are homeomorphic to S^1. An important object in conformal field theory is the **rigged Riemann surface**, which is a Riemann surface Σ^B together with a set of homeomorphisms $\psi_i : \partial_i \Sigma^B \to S^1$. As well as the ordering, each boundary component is labeled as either **incoming** or **outgoing**. Keeping track of this data is not important for our results, so it is subsequently neglected. We denote the rigged Riemann surface by an ordered pair (Σ^B, ψ), where $\psi = (\psi_1, \ldots, \psi_n)$, and the ordered n-tuple of homeomorphisms ψ is referred to as a **rigging**.