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BI-LAGRANGIAN STRUCTURE ON THE SYMPLECTIC AFFINE LIE ALGEBRA OF $\mathbb{R}^2$

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Abstract. In this paper, we give a complete classification of Lagrangian and bi-Lagrangian subalgebras, up to an inner automorphism on $\mathfrak{aff}(2,\mathbb{R})$, and compute the curvatures of some bi-Lagrangian structures.

MSC: 53D05, 53C30
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1. Introduction

The notion of Lagrangian foliations on a symplectic manifold is intimately related to that of geometric quantization in the sense of Kostant-Souriau (see [15] and [17]). On the other hand, the existence of a connection canonically subordinate to a symplectic manifold is an important tool to obtain a formal deformation quantisation introduced by Flato, Lichnerowicz and Sternheimer in [2] and in [8]. An additional structure on the manifold ensures a canonical choice of symplectic connection. A bi-Lagrangian manifold (i.e., a symplectic manifold endowed with two transversal Lagrangian foliations) admits a canonical symplectic connection, which has been introduced by Hess in [11].

A symplectic Lie group (G,ω^+), is a Lie group G equipped with a left-invariant symplectic form ω^+. If we denote by $\mathfrak{g}$ the Lie algebra of G and $\omega=\omega^+(e)$, with e the unit of G, the pair $(\mathfrak{g},\omega)$ is called a symplectic Lie algebra. A symplectic Lie group (G,ω^+) is called a Frobenius Lie group if $\omega^+=\mathrm{d}\alpha^+$, where α^+ is a left-invariant one-form on G. A subalgebra L of $(\mathfrak{g},\omega)$ is called Lagrangian if its dimension is the half of the dimension of $\mathfrak{g}$ and the restriction of ω to L vanishes. A pair (L_1,L_2) of Lagrangian subalgebras is called bi-Lagrangian if $\mathfrak{g}=L_1\oplus L_2$. A symplectic Lie algebra with a pair of Lagrangian subalgebras is called a bi-Lagrangian Lie algebra. Any Lagrangian subalgebra (respectively, bi-Lagrangian pair) defines a Lagrangian foliation (respectively, bi-Lagrangian foliation) on G.

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