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REMARKS ON RIEMANN AND RICCI SOLITONS IN (α, β)-CONTACT METRIC MANIFOLDS

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Abstract. We study almost Riemann solitons and almost Ricci solitons in an (α, β)-contact metric manifold satisfying some Ricci symmetry conditions, treating the case when the potential vector field of the soliton is pointwise collinear with the structure vector field.

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1. Introduction

Riemann and Ricci solitons are generalized fixed points of the Riemann and Ricci flow, respectively. They are defined by a smooth vector field V and a real constant λ which satisfy, respectively, the following equations

$$\frac{1}{2}\pounds_V g \odot g + R = \lambda G \qquad \text{(Riemann soliton)} \tag{1}$$

where $G := \frac{1}{2} g \odot g$, $\pounds_V$ is the Lie derivative operator in the direction of the vector field V, R is the Riemann curvature of g, and

$$\frac{1}{2}\pounds_V g + \mathrm{Ric} = \lambda g \qquad \text{(Ricci soliton)} \tag{2}$$

where Ric is the Ricci curvature of g. The above notation $\odot$ stands for the Kulkarni-Nomizu product, which for two arbitrary $(0, 2)$-tensor fields T_1 and T_2 on M, is defined by

$$\begin{aligned}(T_1 \odot T_2)(X, Y, Z, W) := {} & T_1(X, W)T_2(Y, Z) + T_1(Y, Z)T_2(X, W) \\ & - T_1(X, Z)T_2(Y, W) - T_1(Y, W)T_2(X, Z)\end{aligned}$$

for any $X, Y, Z, W \in \mathfrak{X}(M)$, where $\mathfrak{X}(M)$ is the set of all vector fields on M.

If λ is a smooth function in (1) and (2), then the soliton will be called almost Riemann and almost Ricci soliton, respectively.

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