

## A NOTE ON THE REPRESENTATION OF CLIFFORD ALGEBRAS

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**Abstract.** In this note we construct explicit complex and real faithful matrix representations of the Clifford algebras  $Cl_{p,q}$ . The representation is based on Pauli matrices and has an elegant structure similar to the fractal geometry. In the cases  $p + q = 4m$ , the representation is unique in equivalent sense, and the 1 + 3 dimensional space-time corresponds to the simplest and best case. Besides, the relation between the curvilinear coordinate frame and the local orthonormal basis in the curved space-time is discussed in detail, the covariant derivatives of the spinor and tensors are derived, and the connection of the orthogonal basis in tangent space is calculated. These results are helpful for both theoretical analysis and practical calculation. The basis matrices are the faithful representation of Clifford algebras in any  $p + q$  dimensional Minkowski space-time or Riemann space, and the Clifford calculus converts the complicated relations in geometry and physics into simple and concise algebraic operations. Clifford numbers over any number field  $\mathbb{F}$  expressed by this matrix basis form a well-defined  $2^n$  dimensional hypercomplex number system. Therefore, we can expect that Clifford algebras will complete a large synthesis in science.

*MSC:* 15A66, 15A30, 15B99

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