

SYMBOL CORRESPONDENCE FOR EUCLIDEAN SYSTEMS

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Abstract. The three main objects that serve as the foundation of quantum mechanics on phase space are the Weyl transform, the Wigner distribution function, and the \star -product of phase space functions.

In this article, the \star -product of functions on the Euclidean motion group of rank three, $E(3)$, is constructed. C^* -algebra properties of \star_s on $E(3)$ are presented, establishing a phase space symbol calculus for functions whose parameters are translations and rotations. The key ingredients in the construction are the unitary irreducible representations of the group.

MSC: 81S30, 53D55, 81R05

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