

CONFORMAL MINIMAL FOLIATIONS ON SEMI-RIEMANNIAN LIE GROUPS

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We study left-invariant foliations \mathcal{F} on semi-Riemannian Lie groups G generated by a subgroup K . We are interested in such foliations which are conformal and with minimal leaves of codimension two. We classify such foliations \mathcal{F} when the subgroup K is one of the important groups $SU(2)$, $SL_2(\mathbb{R})$, $SU(2) \times SU(2)$, $SU(2) \times SL_2(\mathbb{R})$, $SU(2) \times SO(2)$, $SL_2(\mathbb{R}) \times SO(2)$. This way we construct new multi-dimensional families of Lie groups G carrying such foliations in each case. These foliations \mathcal{F} produce local complex-valued harmonic morphisms on the corresponding Lie group G . This means that they provide the existence of solutions to a difficult over-determined non-linear system of partial differential equations.

MSC: 58E20, 53C43, 53C30

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