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ANALYTICAL DESCRIPTIONS OF PERSEUS SPIRICS

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A plethora of explicit formulas that parameterize any type of the spiric sections are derived from the first principles.

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1. Introduction

According to Proclus (circa 5-th century AD) – a very influential Greek philosopher and mathematician the ancient Greek geometer Perseus (circa 150 BC) had treated the sections of the spire, or torus (sometimes called an anchor-ring) and subsequently was generally credited with the discovery and characterization of the spiric sections.

By their very definition these curves are the cuts off the tori with the planes that are parallel to their axes of symmetry.

Geminus (circa 100 BC) – another ancient Greek astronomer and mathematician had stated that Perseus wrote an epigram on his discovery: "Three curves upon five sections finding, Perseus made offering to the gods ..."

There many different explications of this epigram have been given. One of them suggests that Perseus found five sections, but only three of them gave new curves. Just these three types of curves will be described in full detail below.

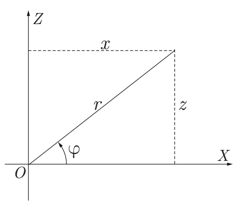

Figure 1. Polar coordinate system.

Let us start with introducing the polar coordinates in the Euclidean plane $XOZ \simeq \mathbb{R}^2$ – see Fig. 1

$$x = r\cos(\varphi), \qquad z = r\sin(\varphi), \qquad r \in \mathbb{R}^+. \tag{1}$$

Relying on them it is easy to describe in explicit form the surface of any particular torus [12, p. 60] of inner radius R and outer radius r, i.e.,

$$\mathbf{x}(\varphi,\psi) = \Big(\big(R + r\cos(\varphi)\big)\cos(\psi),\ \big(R + r\cos(\varphi)\big)\sin(\psi),\ r\sin(\varphi)\Big). \tag{2}$$

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