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HYPERCOMPLEX NUMBERS AND ROOTS OF ALGEBRAIC EQUATION

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By means of hypercomplex numbers, in this paper we discuss algebraic equations and obtain some interesting relations. A structure equation $A^2 = nA$ of a group is derived. The matrix representation of a group constitutes the basis elements of a hypercomplex number system. By a canonical real matrix representation of a cyclic group, we define the cyclic number system, which is exactly the solution space of the higher order algebraic equations, and thus can be used to solve the roots of algebraic equations. Hypercomplex numbers are linear algebras with definition of multiplication and division, satisfying the associativity and distributive law, which provide a unified, standard, and elegant language for many complex mathematical and physical objects. So, we have one more proof that the hypercomplex numbers are worthy of application in teaching and scientific research.

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1. Introduction

The development of number systems had gone through a long and difficult historical process, not as simple as it seems today. Having real numbers, complex numbers and quaternions, a natural idea is to similarly expand the number system by abandoning the algebraic rules as little as possible. But in 1878, an important theorem, proved by the German mathematician F. Frobenius (1849–1917), gave a negative conclusion. The theorem shows that $\mathbb{R}, \mathbb{C}, \mathbb{H}$ are the only finite-dimensional associative division (without zero factors) algebras over $\mathbb{R}$. Subsequently, a generalized Frobenius' theorem showed that if the associative law of multiplication is abandoned, the division algebra without zero factors leaves only octonions or Cayley numbers [7, see Chapters 3, 13, 25 and 32].

If the zero-factor condition $\mathbf{ab} = 0 \Leftrightarrow (\mathbf{a} = 0 \text{ or } \mathbf{b} = 0)$ is relaxed, then many new associative algebras or hypercomplex numbers can be defined by matrix algebra. For example, for any finite group $G = \{g_a\,;\ a = 0, 1, \ldots, n-1\}$, by the matrix