JGSP **64** (2022) 29–37

JOURNAL OF
Geometry and Symmetry
in Physics
ISSN 1312-5192

EXPLICIT SOLUTION OF THE FOCUS LOCUS PROBLEM FOR THE HARMONIC OSCILLATOR ORBITS IN THE PLANE

CLEMENTINA D. MLADENOVA[1] AND IVAÏLO M. MLADENOV[1,2]

Presented by Ivaïlo M. Mladenov

Dynamical orbits of the harmonic oscillator potential in the plane are ellipses which depend on a real parameter. Some time ago in this journal it has been proven by purely geometrical methods that the locus of the focuses of these ellipses are Cassinian ovals.
Here we present several explicit analytic parameterizations of these remarkable curves. Nominally, their forms depend on the magnitude of the initial distance from the center of attraction and the magnitude of the initial velocity.
We have found a few parameterizations in which the roles of the size and shapes can be clearly distinguished.

MSC: 33E05, 34A05, 53A04, 53A17, 70E40
Keywords: Cassinian oval, ellipse of safety, harmonic oscillator, Jacobian elliptic functions

1. Introduction

This paper is based on the considerations of the dynamics of a very simple mechanical systems behind which one can see quite interesting geometrical content. Formally, the system under consideration concerns mechanical oscillations of a point particle which traces various ellipses but the most amazing fact is that the foci of these ellipses trace Cassinian ovals. The size and shapes of these ovals depend on the magnitudes of their initial positions and velocities and this will be clarified in more details in the text.

2. Harmonic Oscillator Orbits

Let us start by introducing the simple free non-damped harmonic oscillator in the plane which dynamics is governed by the differential equations

$$\ddot{\mathbf{x}} + \omega^2 \mathbf{x} = 0 \tag{1}$$